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Synthesis of filters using distributed RC networks

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SYNTHESIS OF FILTERS USING

DISTRIBUTED RC NETWORKS

BY

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THESIS

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ABSTRACT

This paper deals with the problem of realizing filter and delay functions by means of distributed RC devices. The method is made possible through the use of a distributed RC network which possesses rational admittance parameters.

The technique of approximating transfer functions using polynomial factors that are rational in $e^{a\sqrt{s}}$ is explained. These factors are used to approximate the magnitude and phase characteristics for a low-pass filter. This filter is then realized using the admittance functions studied by Wyndrum,[4] the "$s \rightarrow W$" transformation,[10] and Richard's theorem.[11]

Those factors are then used to approximate high-pass and band-pass filter characteristics. The high-pass filter is realized using a ladder of one-port URCO and URCS networks, and the band-pass filter is realized using the "cascade and stub" technique as explained by Wyndrum.[4]

Finally the use of distributed RCR networks is mentioned and the magnitude and phaseshift of two compensators are plotted.

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LIST OF SYMBOLS

a .... thickness of the dielectric (m)

A_v ... filter voltage transfer function

C .... capacitance (F)

db ... decibel

G_B... conductance (mhos)

L ... length of the distributed RC Line segment (m)

p' ... complex frequency in the p'-plane (Sec^{-1})

R .... resistance (Ω)

s .... complex frequency in the s-plane (sec^{-1})

Y .... admittance (mhos)

y_{21}... short-circuit transfer admittance (mhos)

y_{22}... short-circuit driving point admittance (mhos)

Z .... impedance (Ω)

z_{12}... open-circuit transfer impedance (Ω)

z_{11}... open-circuit driving point impedance (Ω)

ω .... angular frequency (rad/sec)

W.... complex frequency in the W-plane (Sec^{-1})

τ .... RC product (sec)

exp .. exponential

π ... product

Arg .. phase (degrees)

$| \; |$... absolute value

CHAPTER I

INTRODUCTION

Filter and delay networks of the RC type present an attractive solution to the minaturization problem in one area of circuit design. Miniaturization through elimination of the inductor, however, has resulted in a comparative increase in the complexity of the resulting filter and delay networks. This complexity, of course, is a result of the greater number of required components and their associated interconnections. An answer to the complexity problem can be found in the application of distributed parameter devices. In general, distributed networks lend themselves to miniaturization and can replace a sizable number of discrete components.

A synthesis technique for filters using distributed parameter devices will be discussed in this paper. Chapter II will be a review of the previous work done in this field. In Chapter III the magnitude and phase characteristics for polynomial factors which are functions of $e^{a\sqrt{s}}$ will be given and will be used in the following chapters for the approximation of filter transfer functions. In Chapter IV general admittance functions for low-pass filters will be studied and one of those filters will be designed. This network is a cascade of distributed RC realizations which is realized by the use of Richard' s theorem. A delay network which uses a symmetrical lattice of one-port URCO and URCS networks* will be discussed. Chapter V will present a high-pass ladder filter that is composed of the same networks named above. Chapter VI will deal

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*URCO and URCS one-ports are uniform distributed RC line two ports for which port two has been open-circuited and short-circuited respectively.

with a band-pass filter. The distributed RCR networks will be mentioned in Chapter VII where two transfer functions will be approximated.

CHAPTER II

REVIEW OF THE LITERATURE

W. M. Kaufman[1] made a study of the uniform distributed RCR network (URCR) and compared it with the lumped parameter Twin-T network finding the first more useful in microelectronics. He found out that it could be used satisfactorily in high Q tuned amplifiers, oscillators, and threshold transducers.

M. J. Hellstrom[2] proved that the distributed RC ladder networks, which are electrically symmetrical, can be identified by the characteristic that the distributed series resistance is proportional to the distributed shunt capacitance. He also derived chain parameters for those networks.

W. W. Happ, P.S. Castro, and W.D. Fuller[3] applied the use of subnetworks to the synthesis of multiterminal networks.

R. W. Wyndrum[4] presented the first mathematical procedures which may be used to synthesize a prescribed admittance magnitude function with distributed RC networks, giving realizability conditions and a general synthesis procedure. He tested his theoretical work by designing a low-pass filter as a cascade of distributed RC segments. The author's work will be an extension of Wyndrum's. In another paper Wyndrum[5] investigated the frequency response and monolithic embodiment of a distributed RC null network, as Kaufman[1] did, but he added one degree of freedom which allows control of the notch frequency independently of the distributed RC network.

R. A. Dell, Jr., and S. L. Hakimi[6] studied the effect of leakage conductance through the dielectric in distributed RC networks finding that if the leakage is allowed, the cut-off properties of low-pass filters are improved. Networks with this property are referred to as RCG networks.

Richard P. O' Shea[7] discussed the realizability conditions of a driving point impedance as a function of the frequency "$P = \cosh \sqrt{sRC}$". He also applied those conditions to the synthesis of a transfer function using the ladder structure.

Donald G. Barker[8] discussed the use of thin film distributed parameter networks in the synthesis of active filters, presenting a general synthesis technique which can be used in the realization of rational filter and delay functions, he illustrated his technique through the detailed realization of a simple low-pass filter.

T. N. Rao and R. W. Newcomb[9] discussed the synthesis of lumped-distributed RC n-port networks, giving realizability conditions and explaining that with a small change the same ideas could be applied to the synthesis of lumped-distributed LC networks.

CHAPTER III

THE APPROXIMATION PROBLEM

A network designer usually begins with a graphical magnitude function plotted versus radian frequency ω. The function might be either an immittance or a gain function. The gain function is just a quotient of immittance functions so it is sufficient to consider the immittance functions and then apply the results obtained to gain functions.

Wyndrum[4] showed that uniform distributed RC immittance functions are given by hyperbolic functions of $\sqrt{s}$, so it is important for the designer to be able to translate the graphical specifications directly into the hyperbolic functions of $\sqrt{s}$ so that realizability conditions for the distributed networks may be applied.

Consider the form of hyperbolic approximation for a given magnitude-frequency specification as detailed in equation (1). Realizable driving point immittance functions are subclasses[4] of

$$H(s) = \frac{(1 \pm \exp(a\sqrt{s})) \prod_{1}^{n} N_i(s)}{E(s)\,(1 \mp \exp(a\sqrt{s}))\,(1 + \exp(a\sqrt{s}))^{p} \prod_{1}^{m} D_i(s)} \tag{1}$$

where

$$N_i(s) = (\exp(2a\sqrt{s}) + B_i \quad (\exp(a\sqrt{s})) + 1)$$

and

$$D_i(s) = (\exp(2a\sqrt{s}) + A_i \quad (\exp(a\sqrt{s})) + 1)$$

B_i and A_i are real

$$|B_i| < 2, \quad |A_i| < 2$$

p = positive integer

and when*

$F(s) = \sqrt{s}$, $H(s)$ = Impedance

$F(s) = 1/\sqrt{s}$, $H(s)$ = Admittance

$F(s) = 1$, $H(s)$ = Gain function

The "exponential polynomial" factors in (1) possess practical well-behaved magnitude functions. Accordingly, a procedure analogous to the straightline "Bode approximation" for lumped networks is used.

Exponential polynomials provide the vehicle to approximate graphical specifications of real frequency functions directly by analytic expressions in the original plane of definition.

Fig. 3-1 to 3-27 present the real frequency magnitudes and phaseshifts of the exponential factors of interest (even though these polynomial factors are not minimum phase functions, it is desirable to determine the principal angle in order to observe the correspondence between the phase relationship that can be obtained by using distributed RC networks or lumped networks). The left ordinates represent the magnitude of the factors in "db", the right ordinates represent the phase of those polynomials in degree and the abscissas represent the normalized frequency $a^2\omega/2$.

Since the functions to be realized involve products and quotients of such exponential factors, the magnitudes in "db" and the phases of the factors need

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*see reference 4.

only be added graphically to produce the desired RC specification.

To illustrate this approximation technique suppose that a gain function magnitude is specified to be exactly zero at zero frequency, to be at least -25 db for $\omega < 0.13$ and unity for $\omega \geq 1.3$. In order for the magnitude to be zero at zero frequency a term of the form $(\exp(a\sqrt{s}) - 1)$ is needed in the numerator. That term must be cancelled at high frequencies so the factor $(\exp(a\sqrt{s}) + 1)$ must be in the denominator. Two other terms are chosen so that for $\omega < 0.13$ an attenuation of at least 25 db will be obtained. The final form of the gain function is shown in equation (2)

$$A_v(s) = \frac{(\exp(a\sqrt{s}) - 1)\ (\exp(2a\sqrt{s}) - 1.7\ (\exp\ (a\sqrt{s})\ )\ +\ 1)}{(\exp(\ a\sqrt{s})\ +1)\ (\exp\ (2a\sqrt{s})\ +\ 1)} \tag{2}$$

The specified magnitude and the calculated one are shown in figure 3-28.

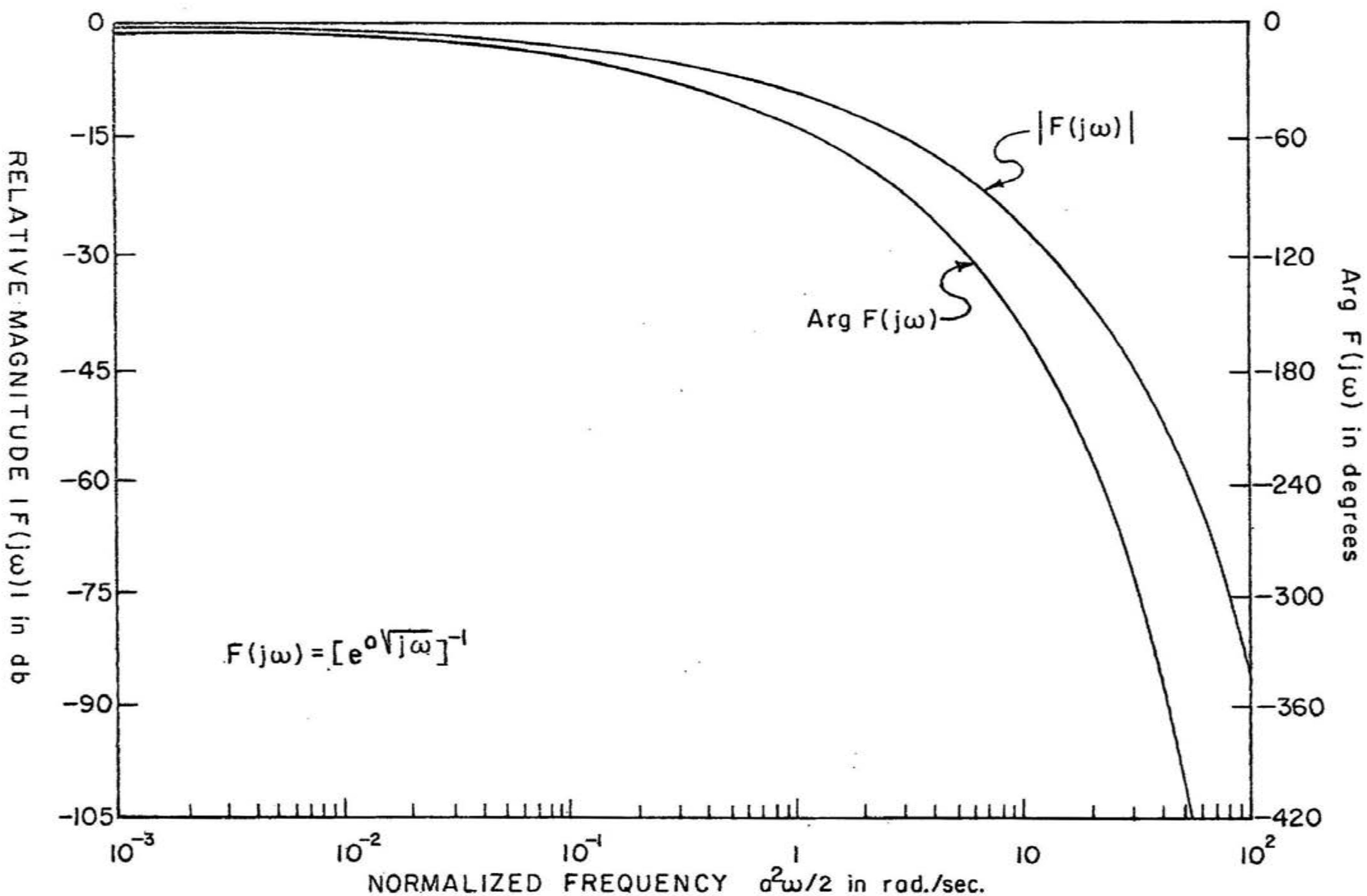


Figure No. 3-1 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RC SYNTHESIS.

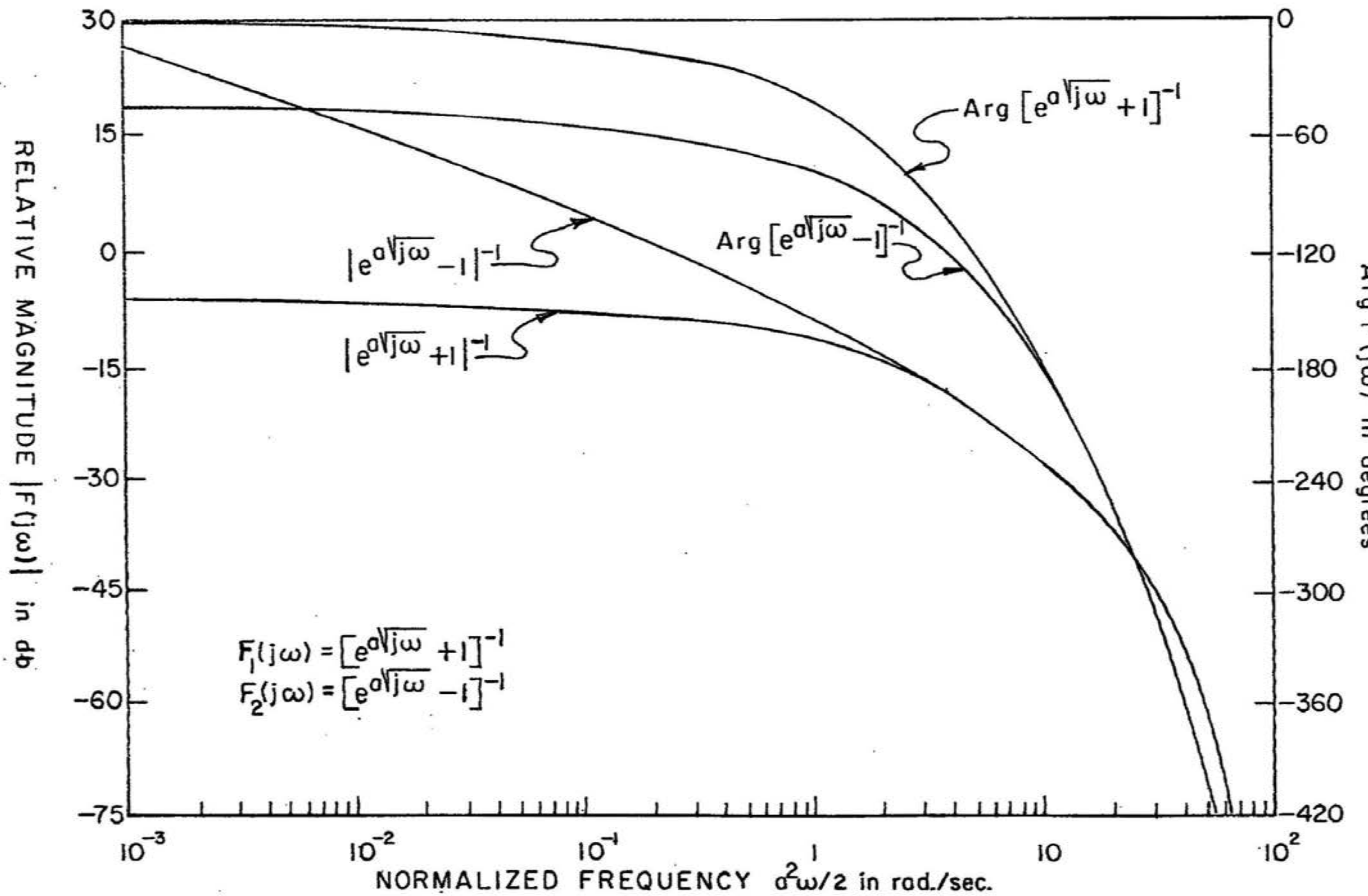


Figure No. 3-2 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RC SYNTHESIS

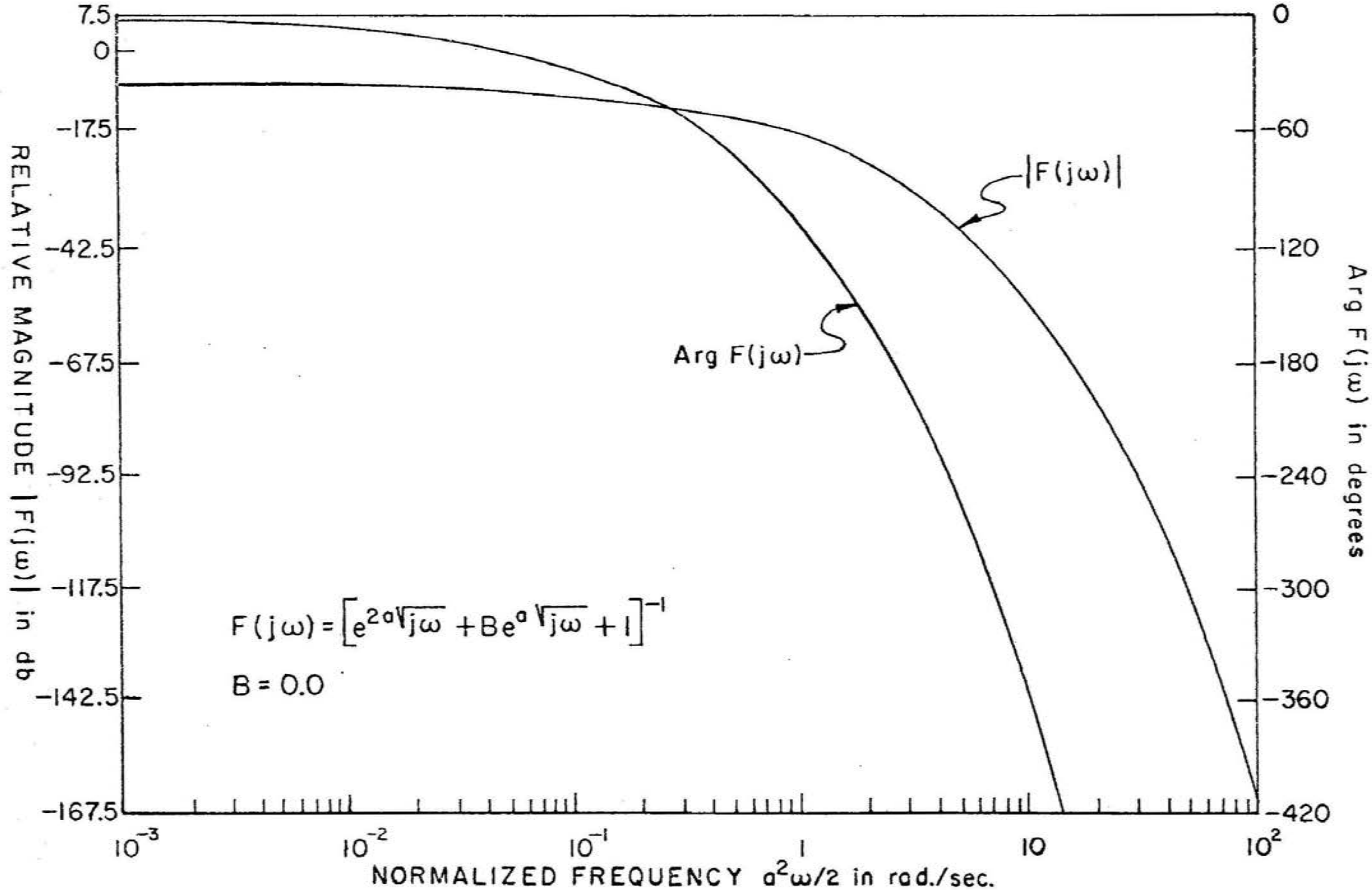


Figure No. 3-3 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RC SYNTHESIS

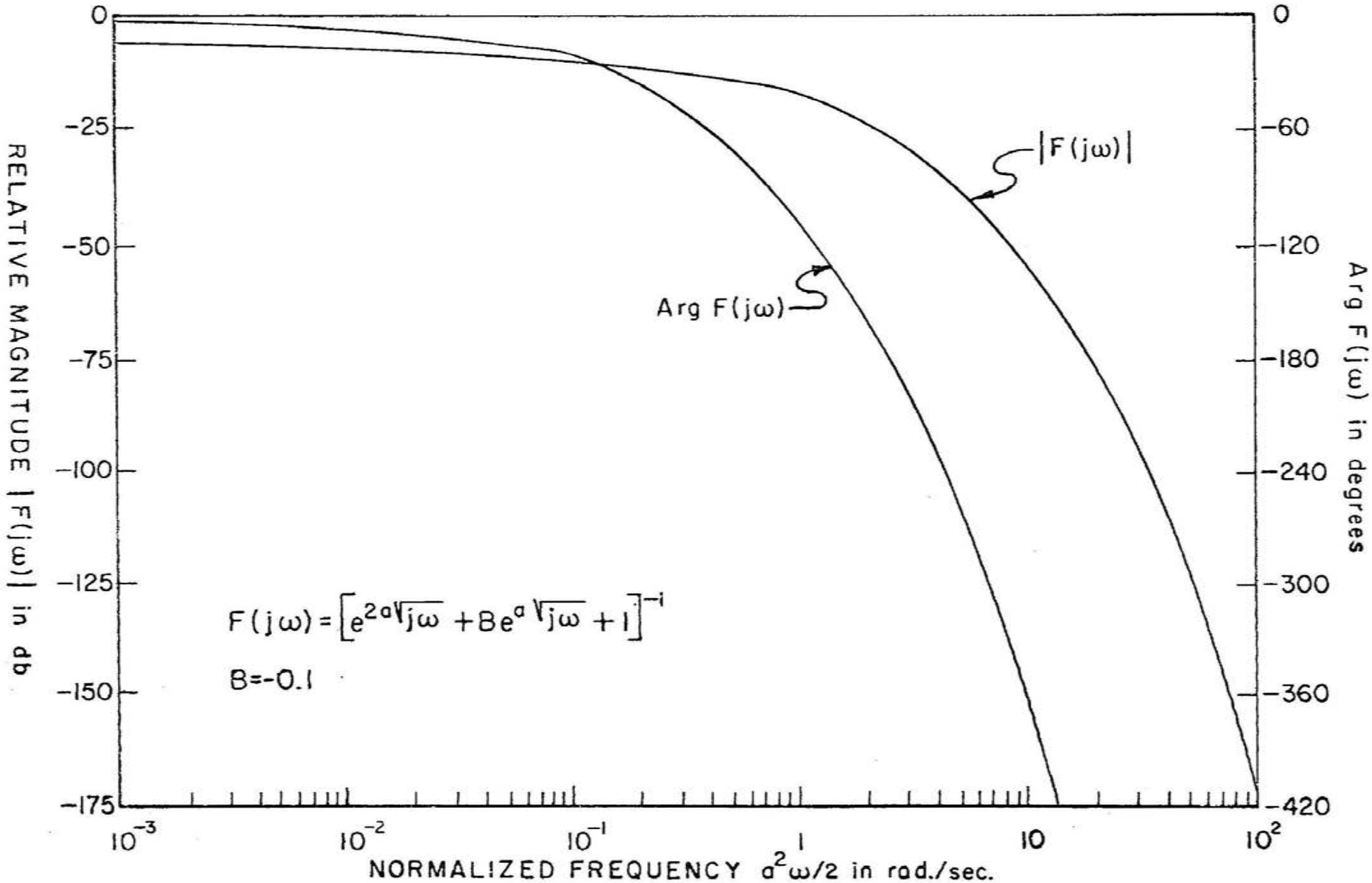


Figure No. 3-4 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RC SYNTHESIS

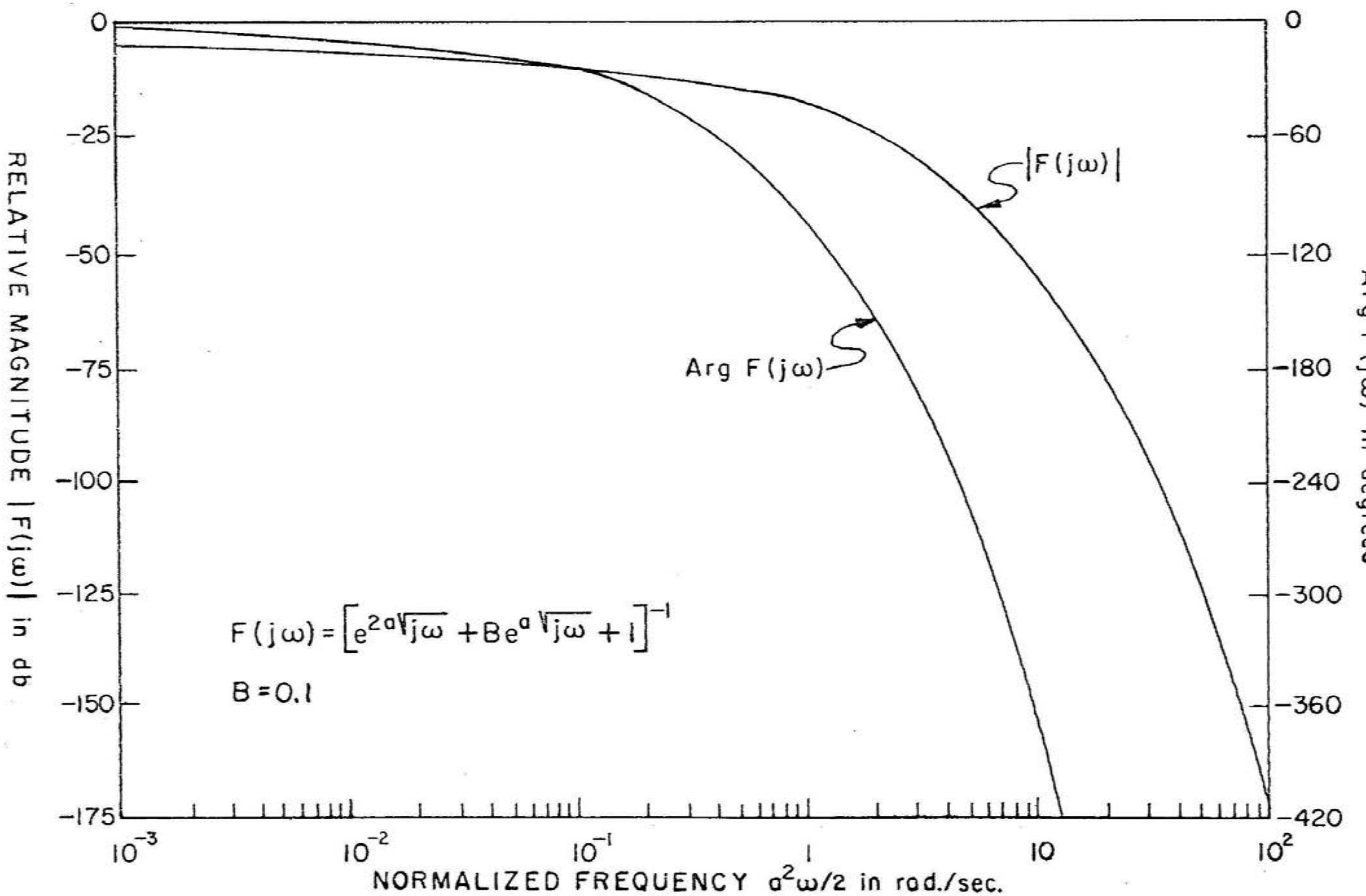


Figure No. 3-5 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RC SYNTHESIS

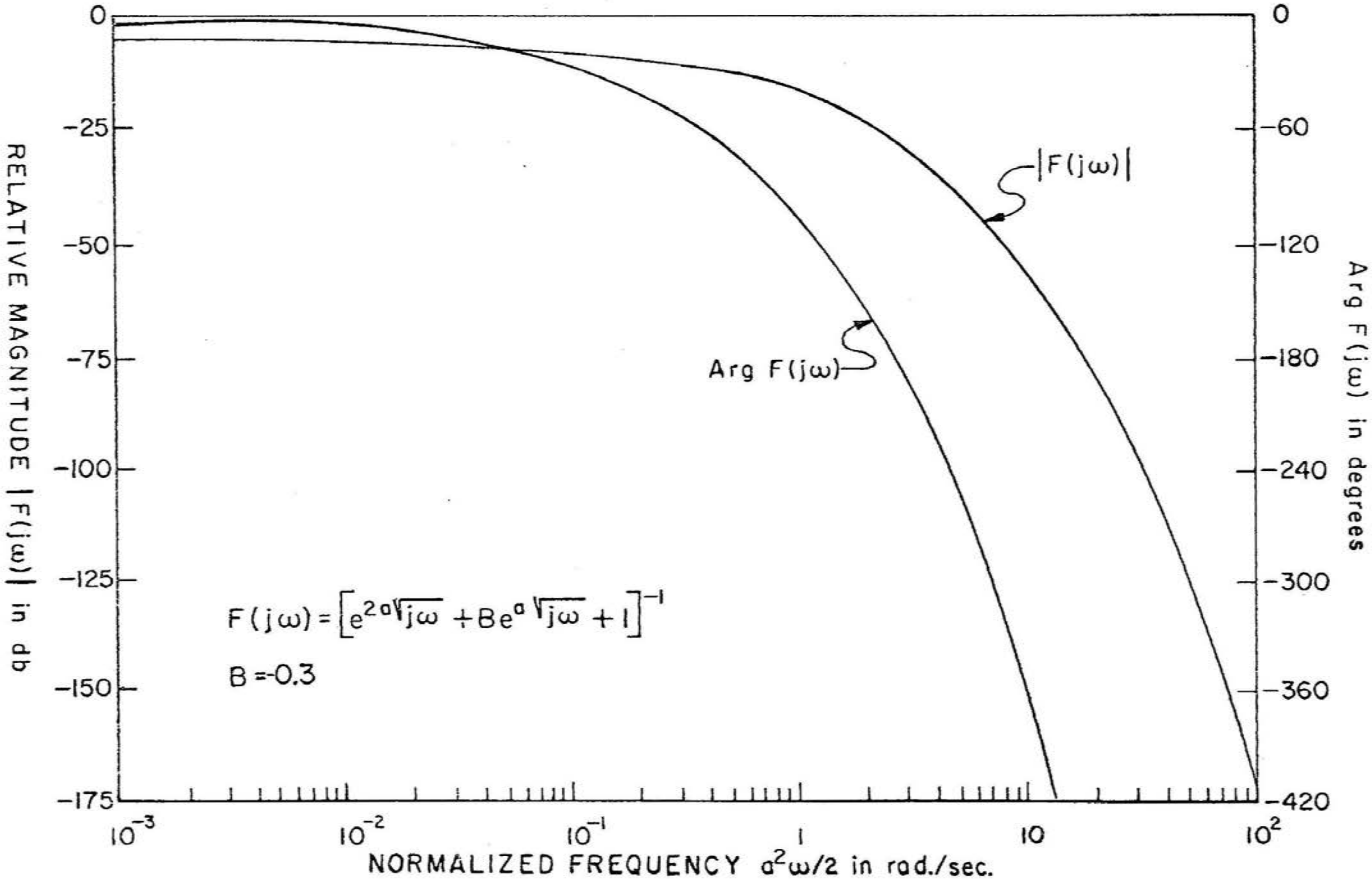


Figure No. 3-6 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RC SYNTHESIS

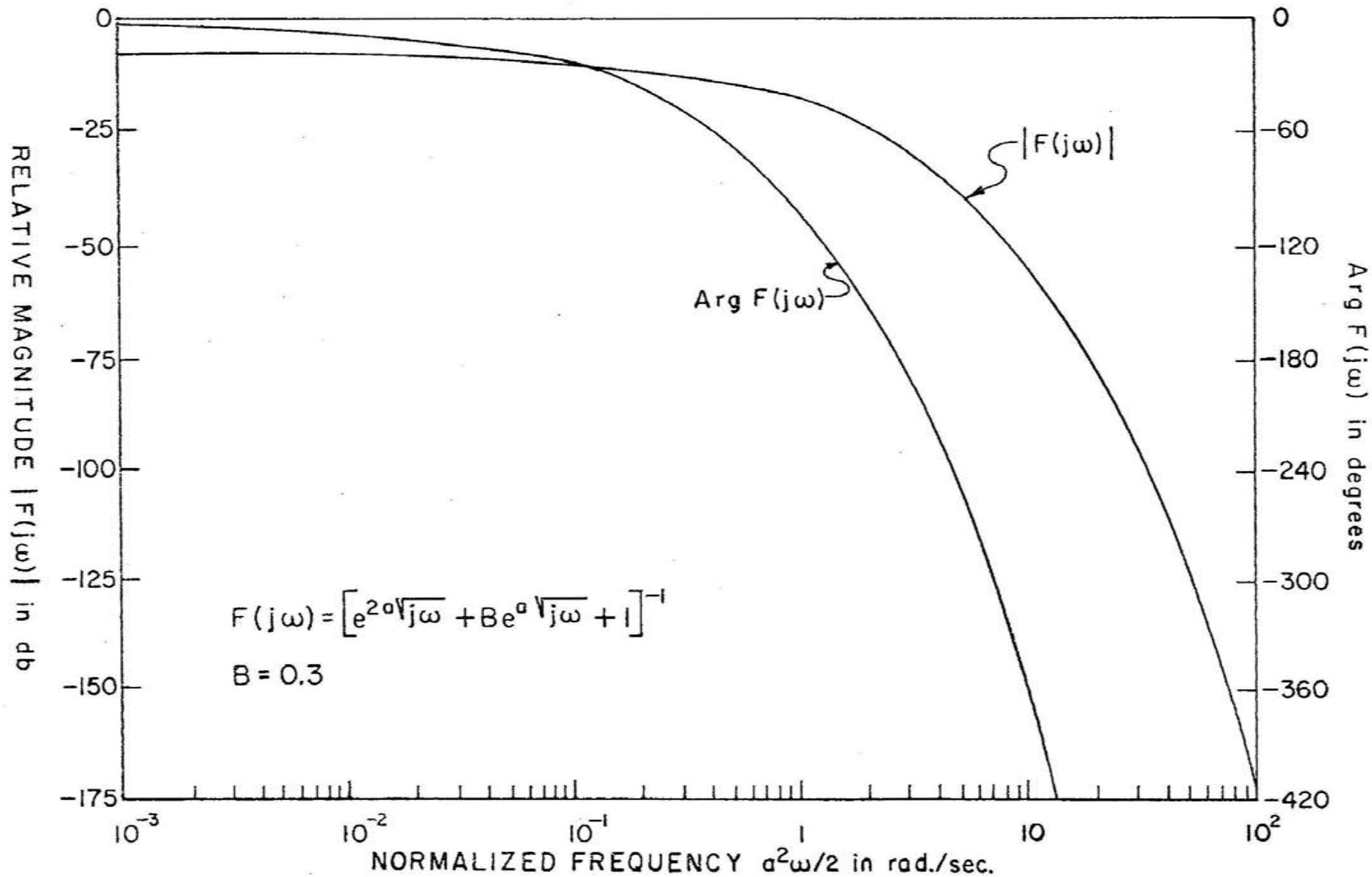


Figure No. 3-7 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RC SYNTHESIS

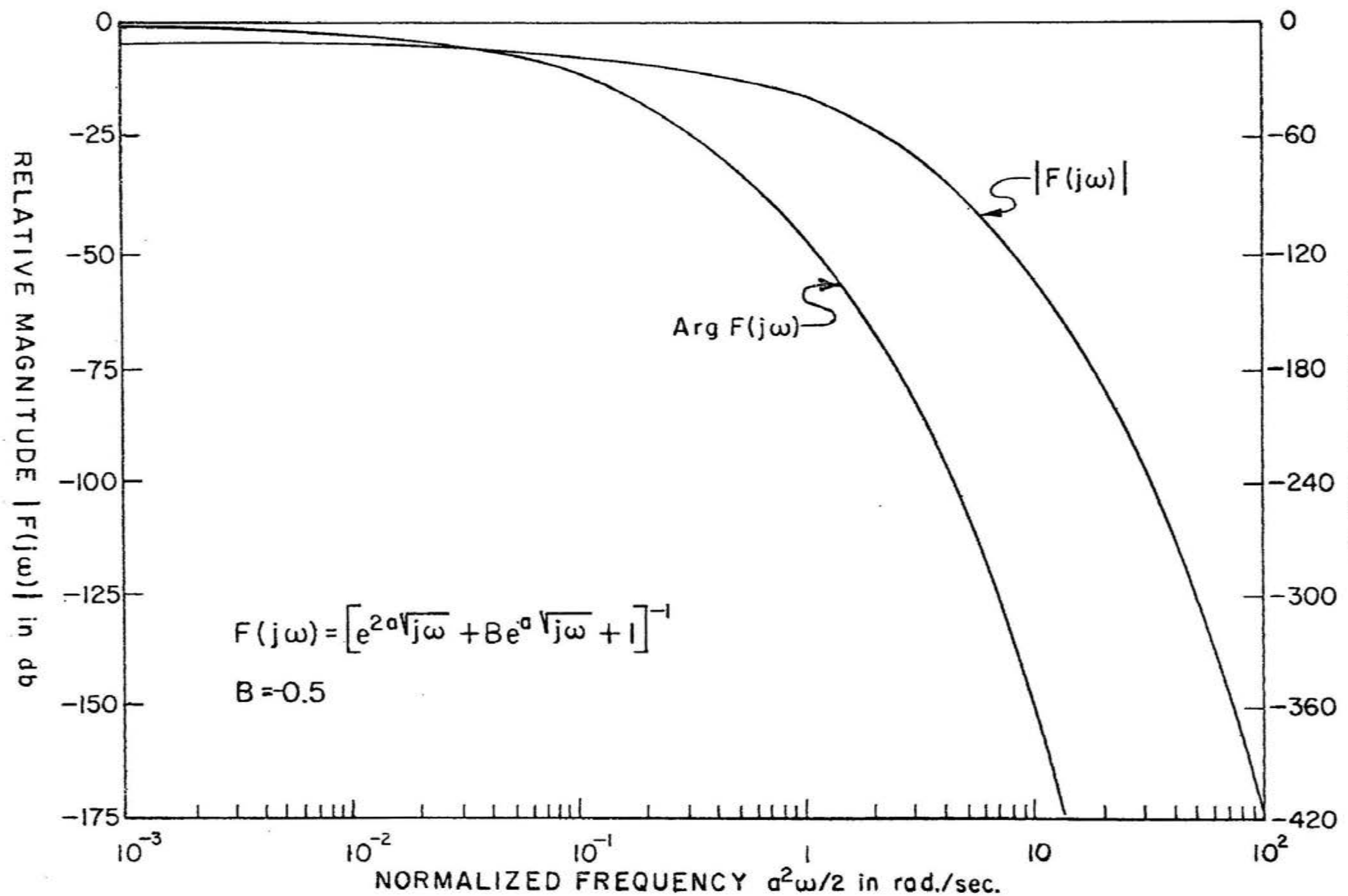


Figure No. 3-8 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RC SYNTHESIS

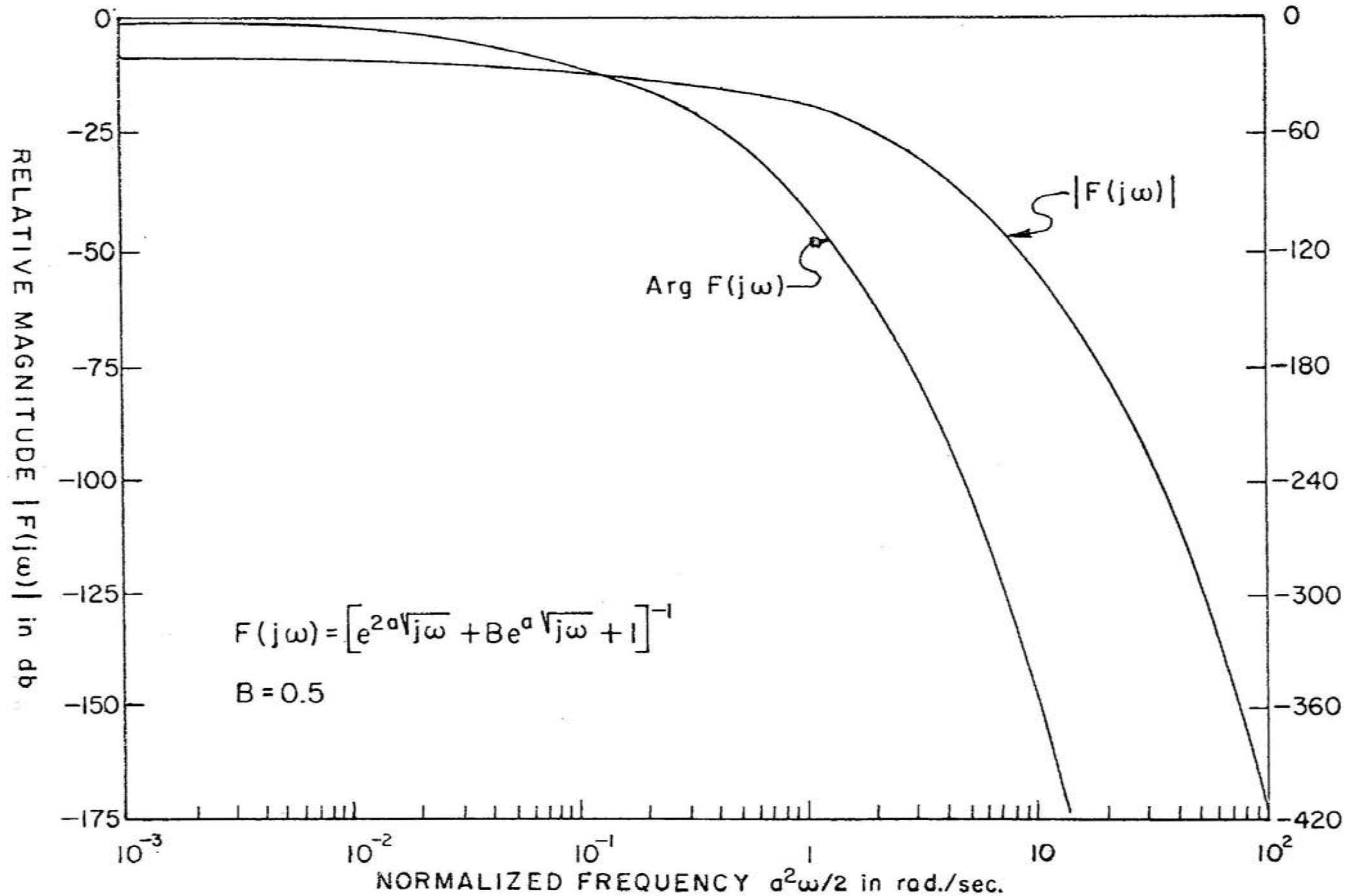


Figure No. 3-9 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RC SYNTHESIS

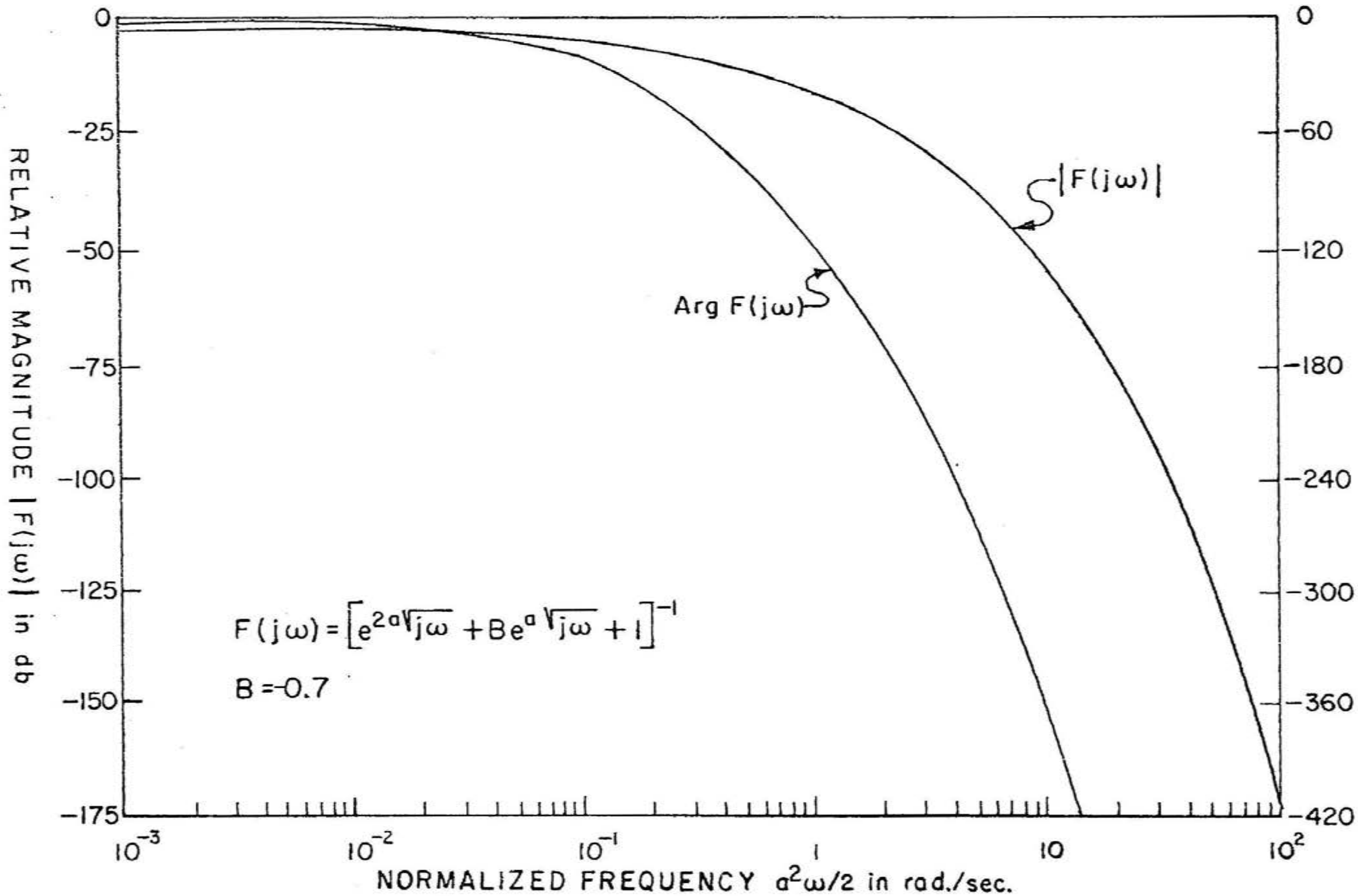
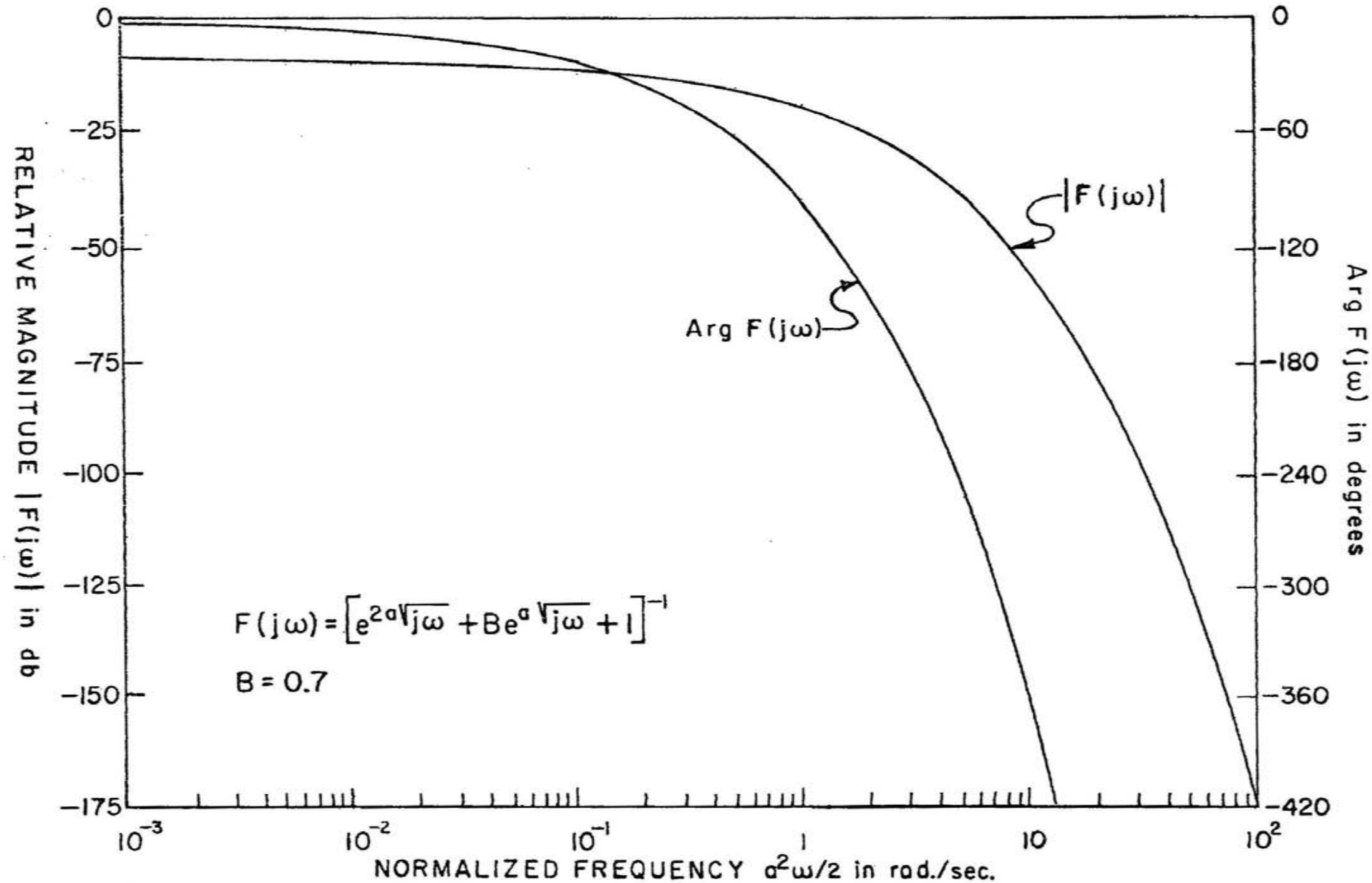


Figure No. 3-10 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RC SYNTHESIS



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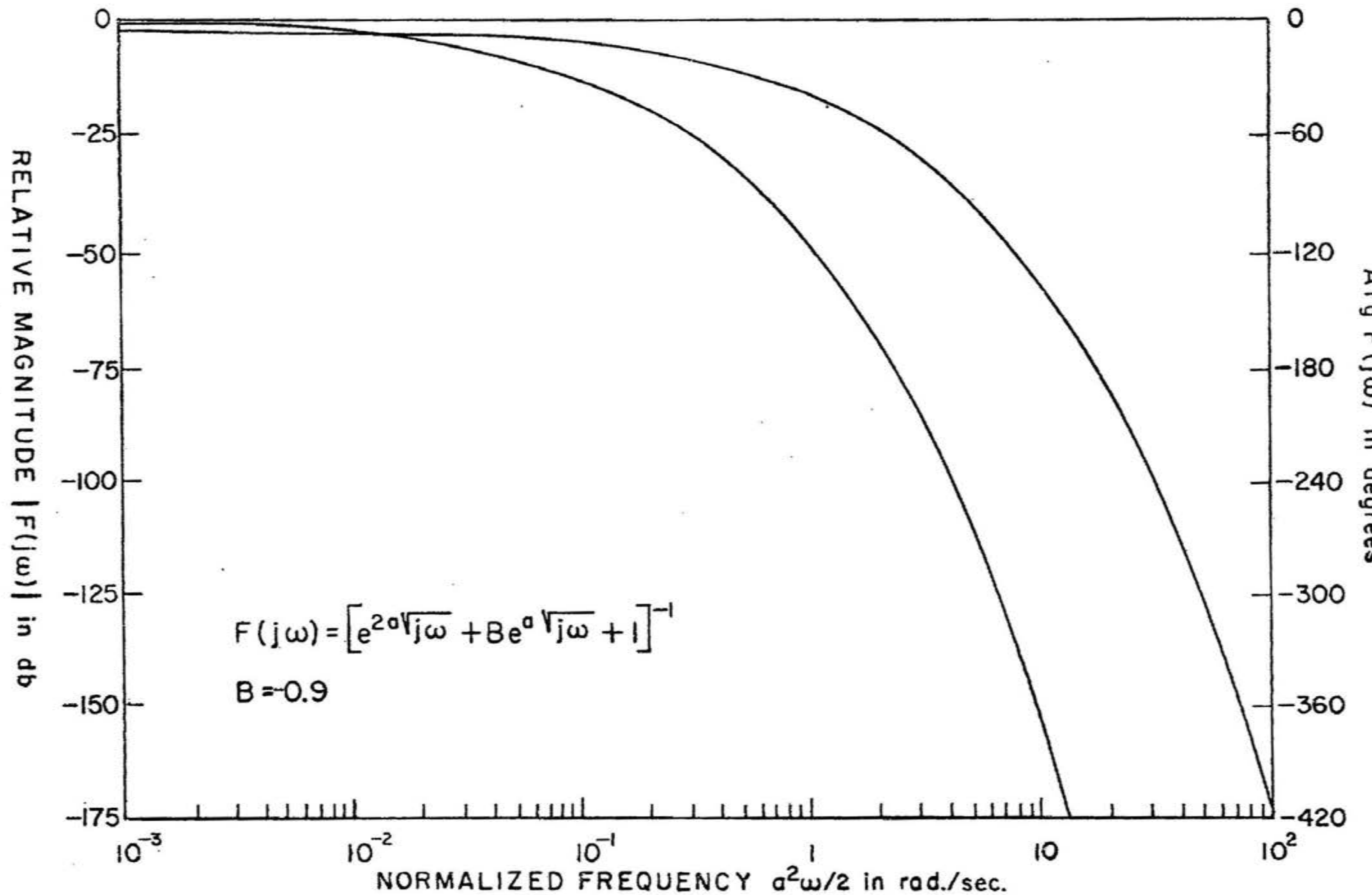

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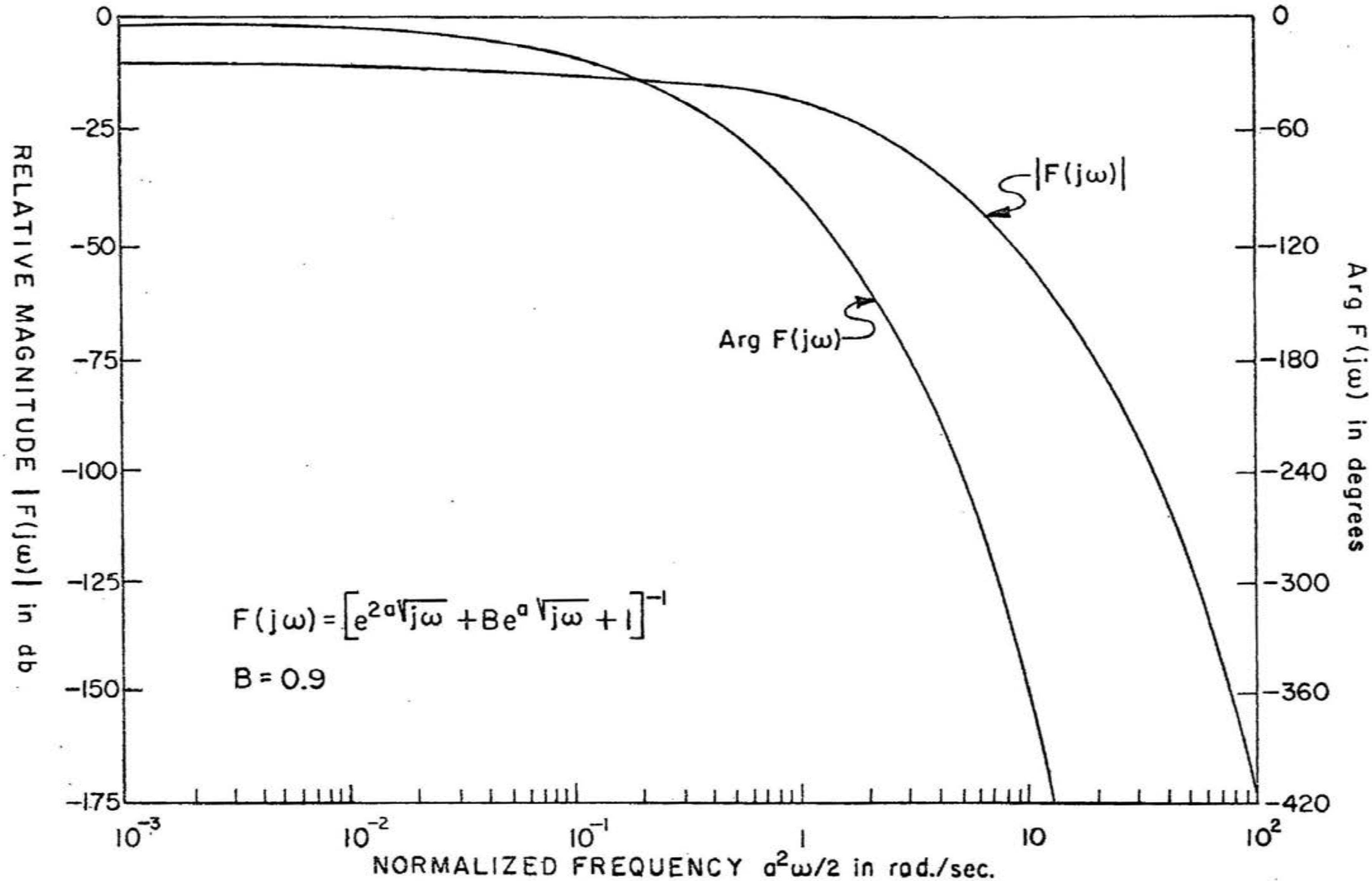


Figure No. 3-13 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RC SYNTHESIS

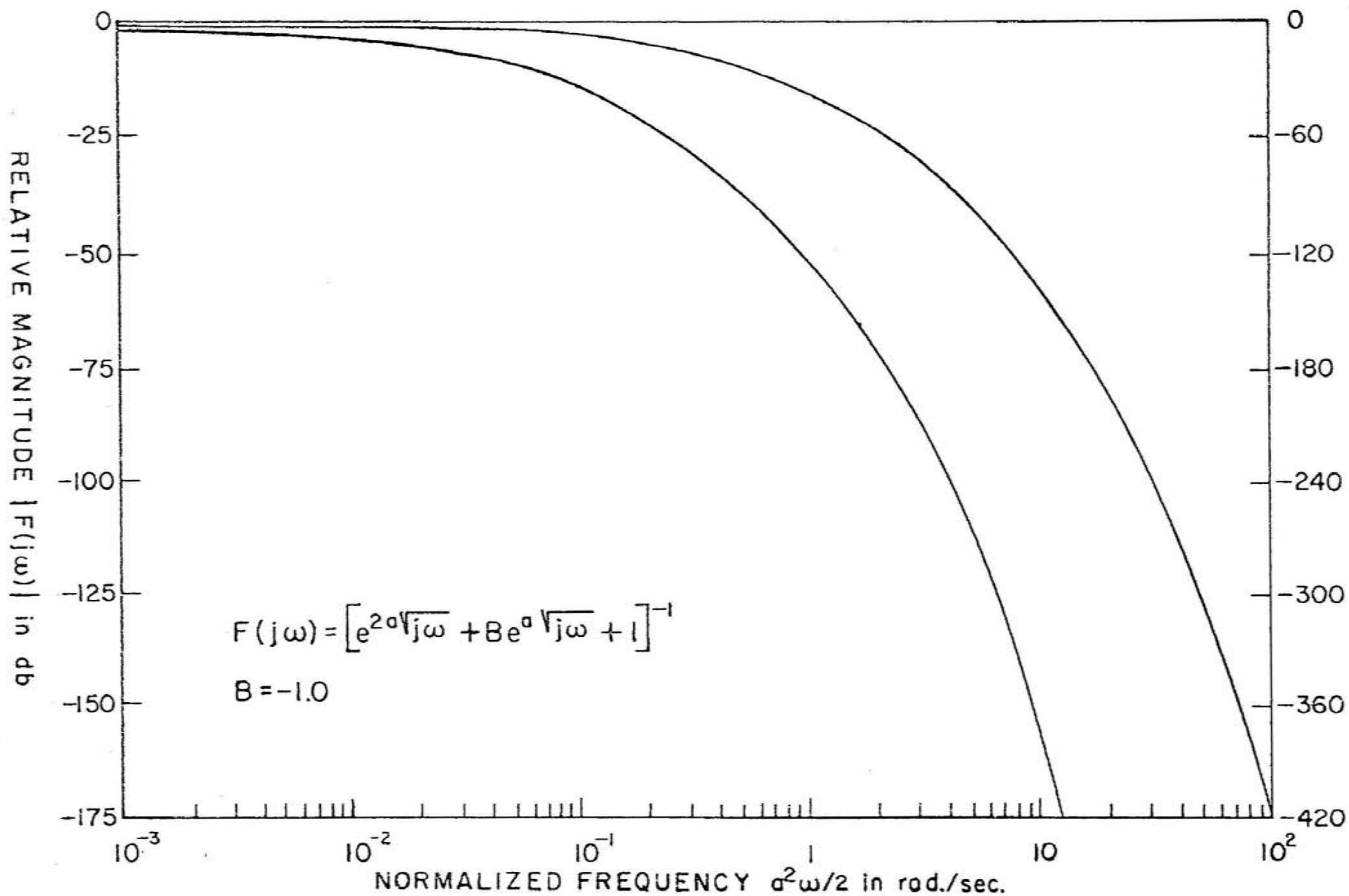
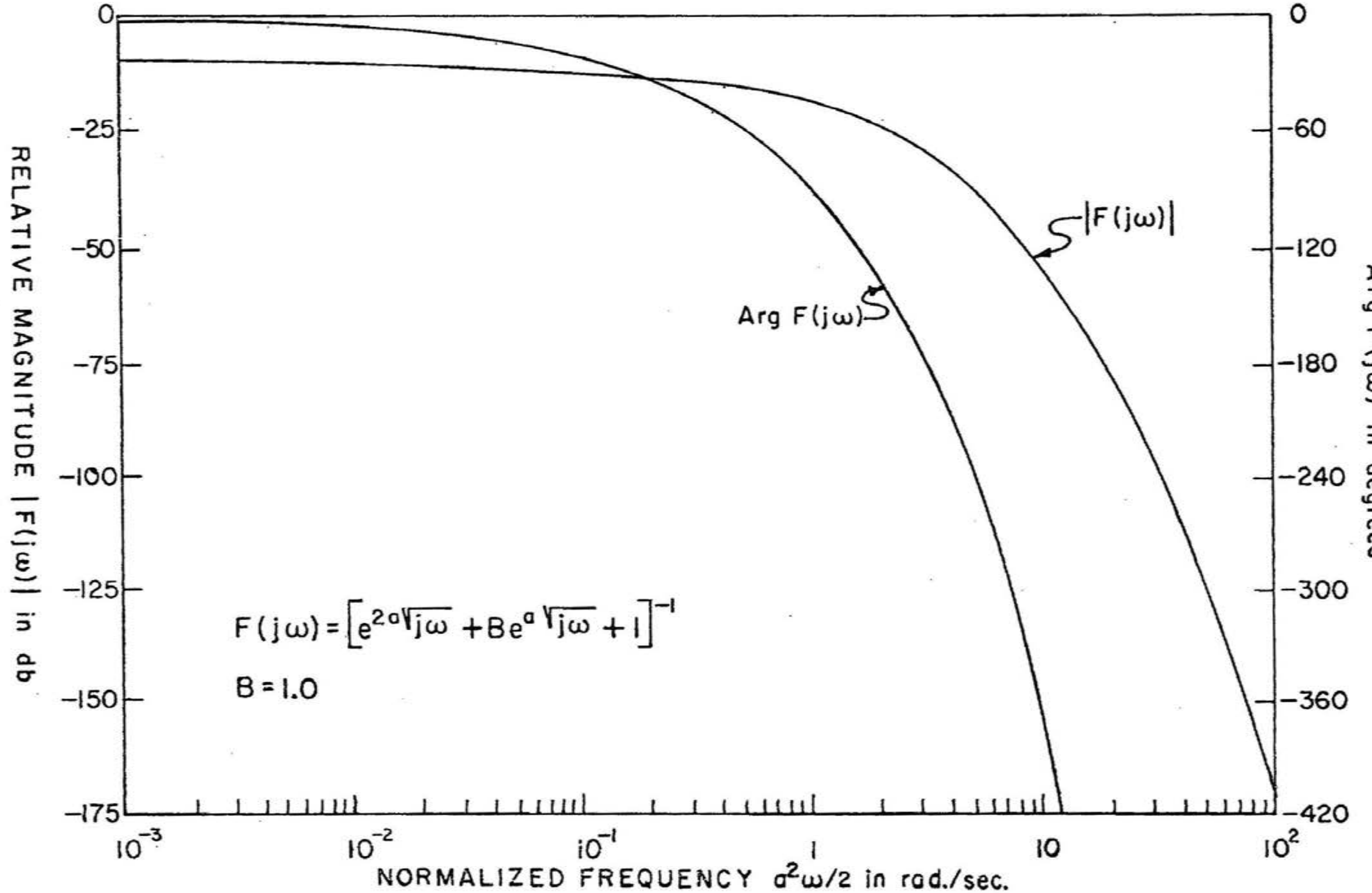


Figure No.3-14 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RC SYNTHESIS



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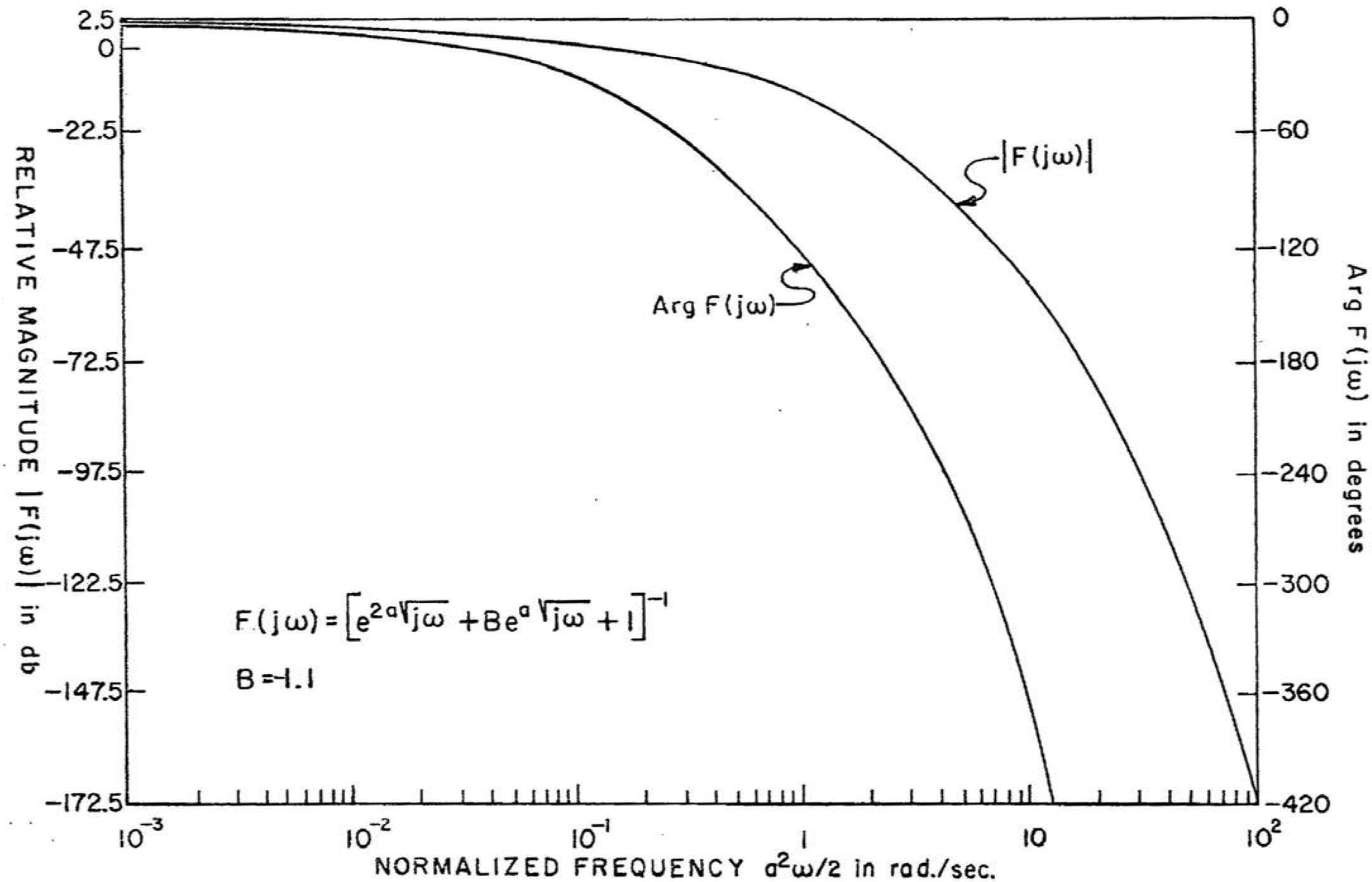

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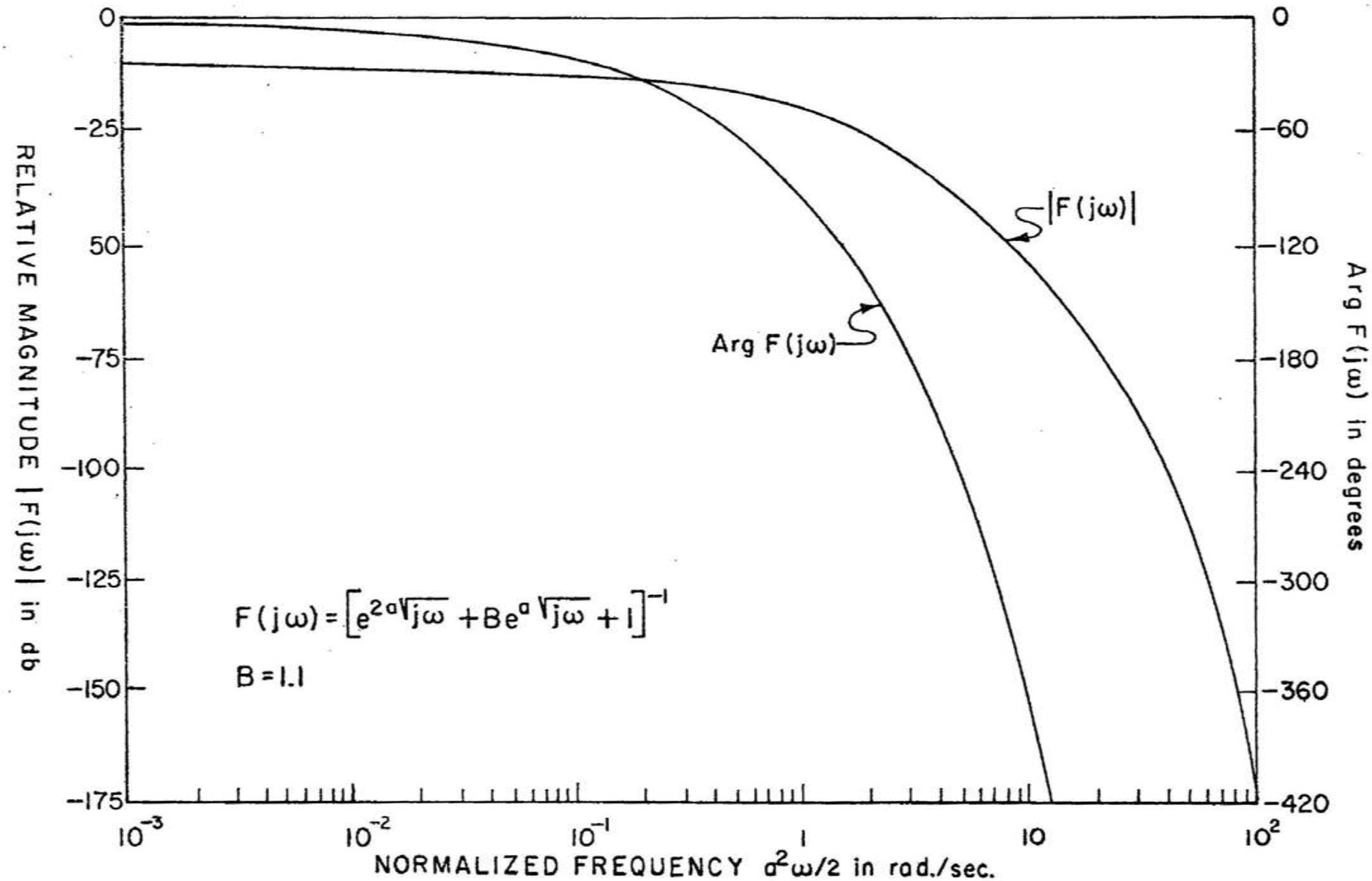


Figure No. 3-17 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RC SYNTHESIS

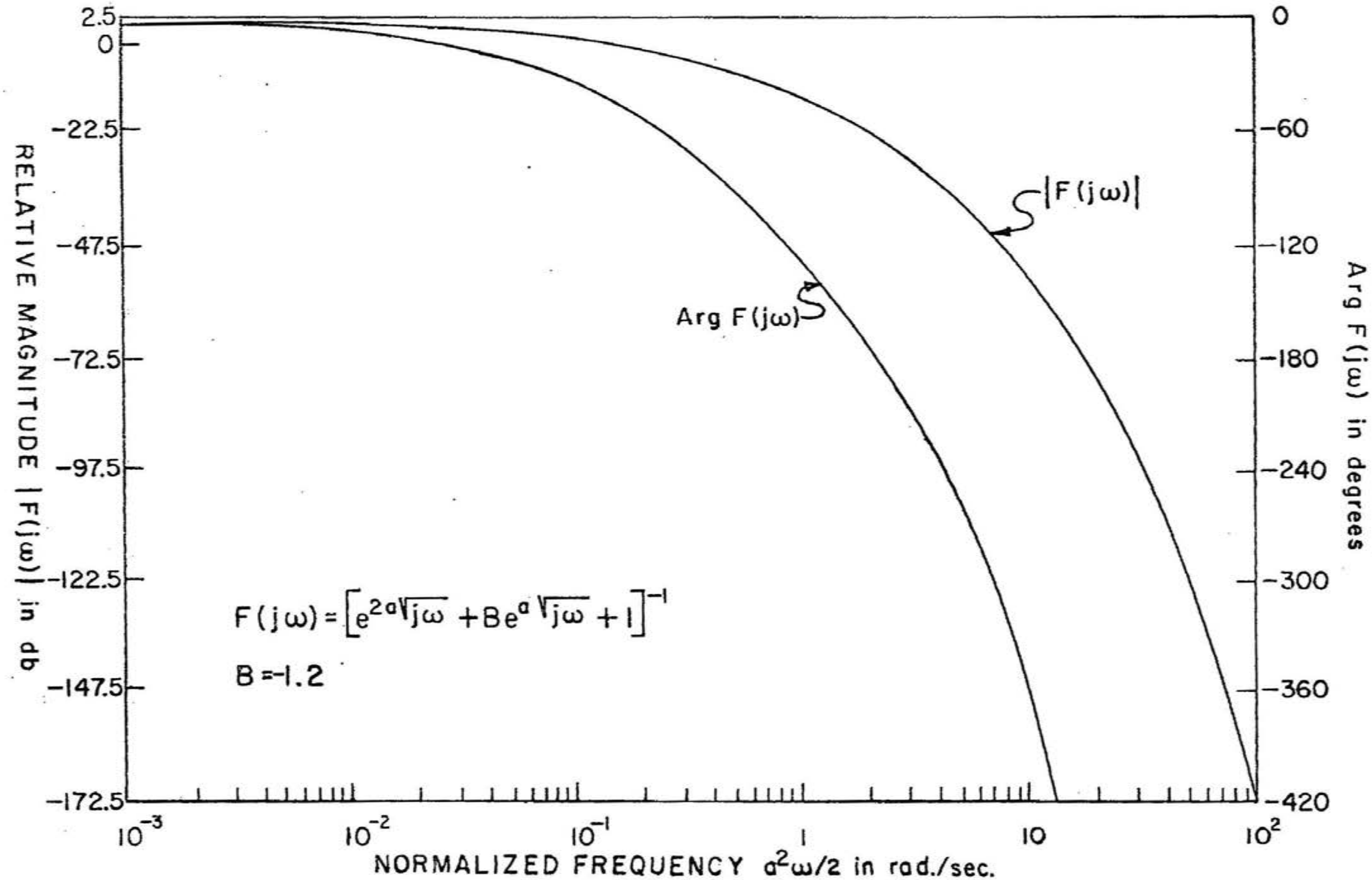
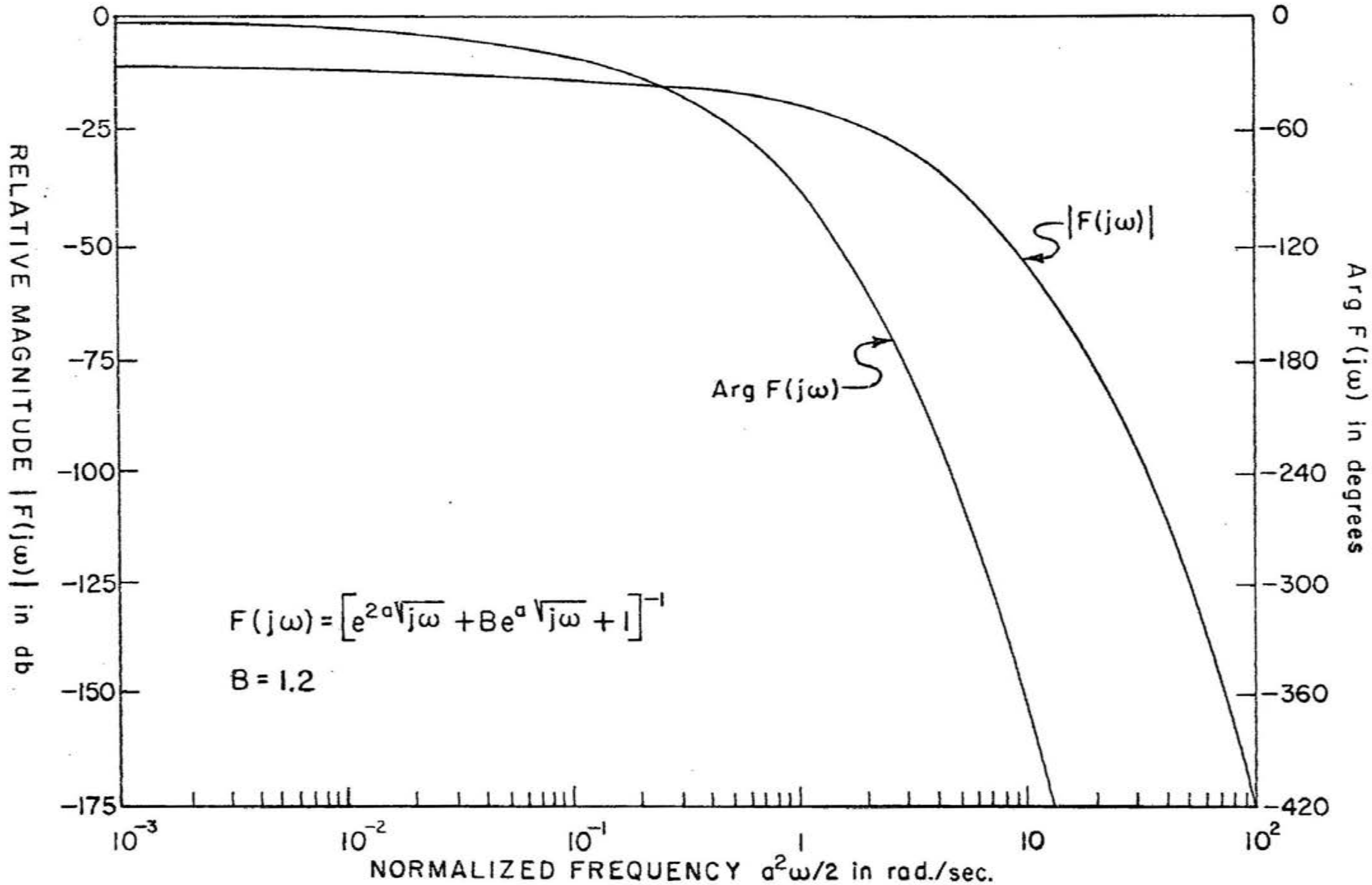


Figure No. 3-18 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RC SYNTHESIS



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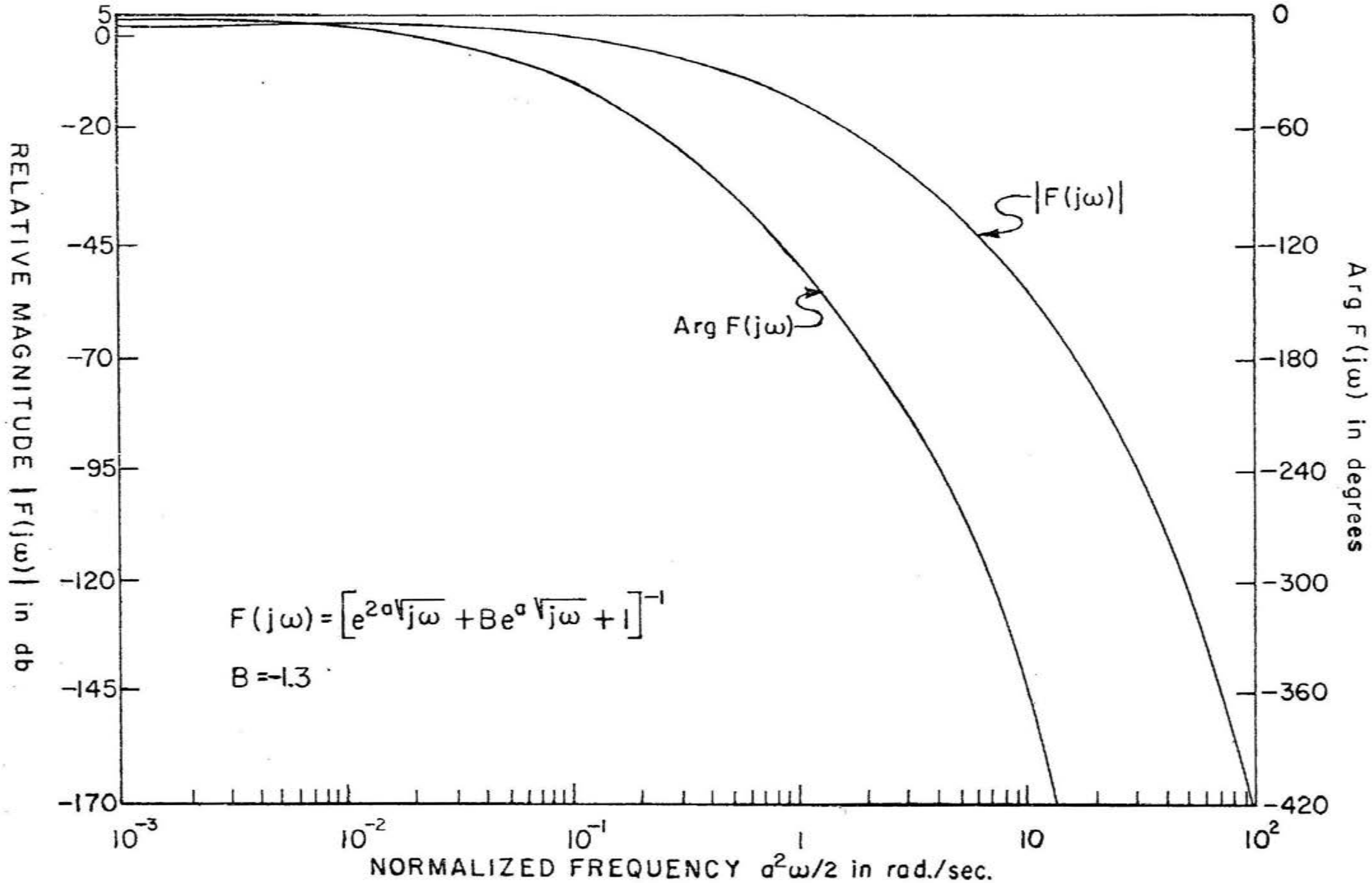

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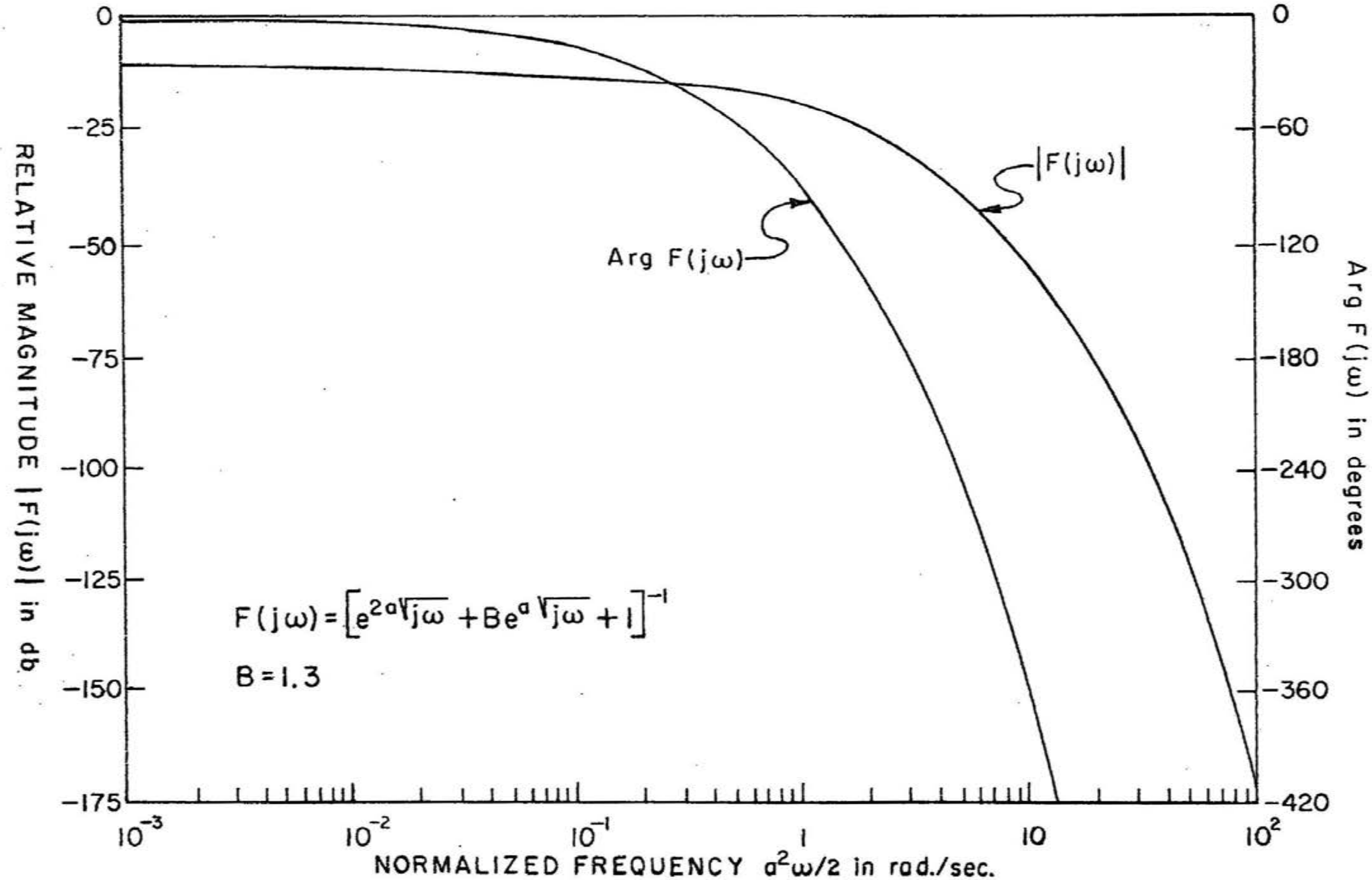

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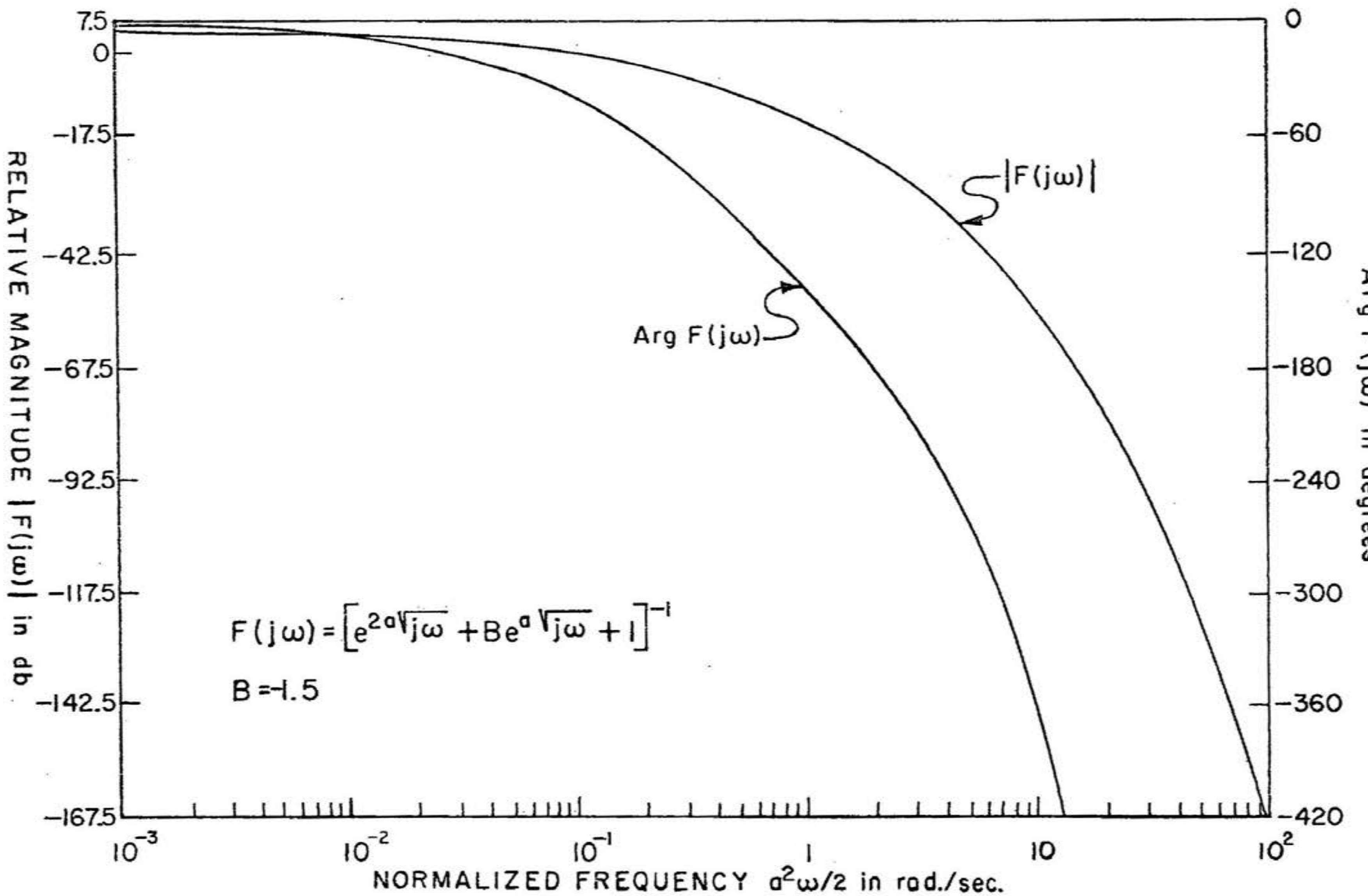

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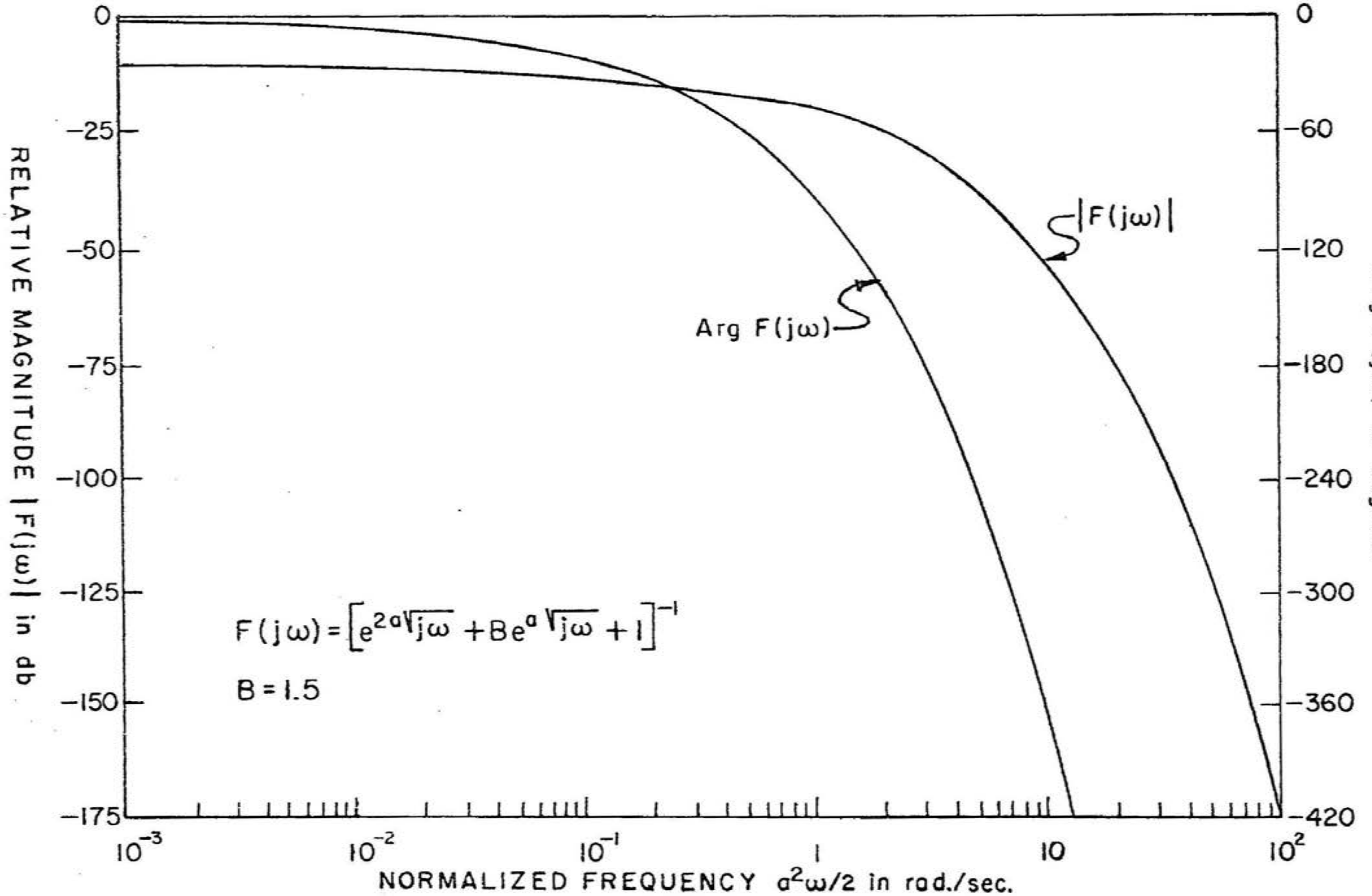

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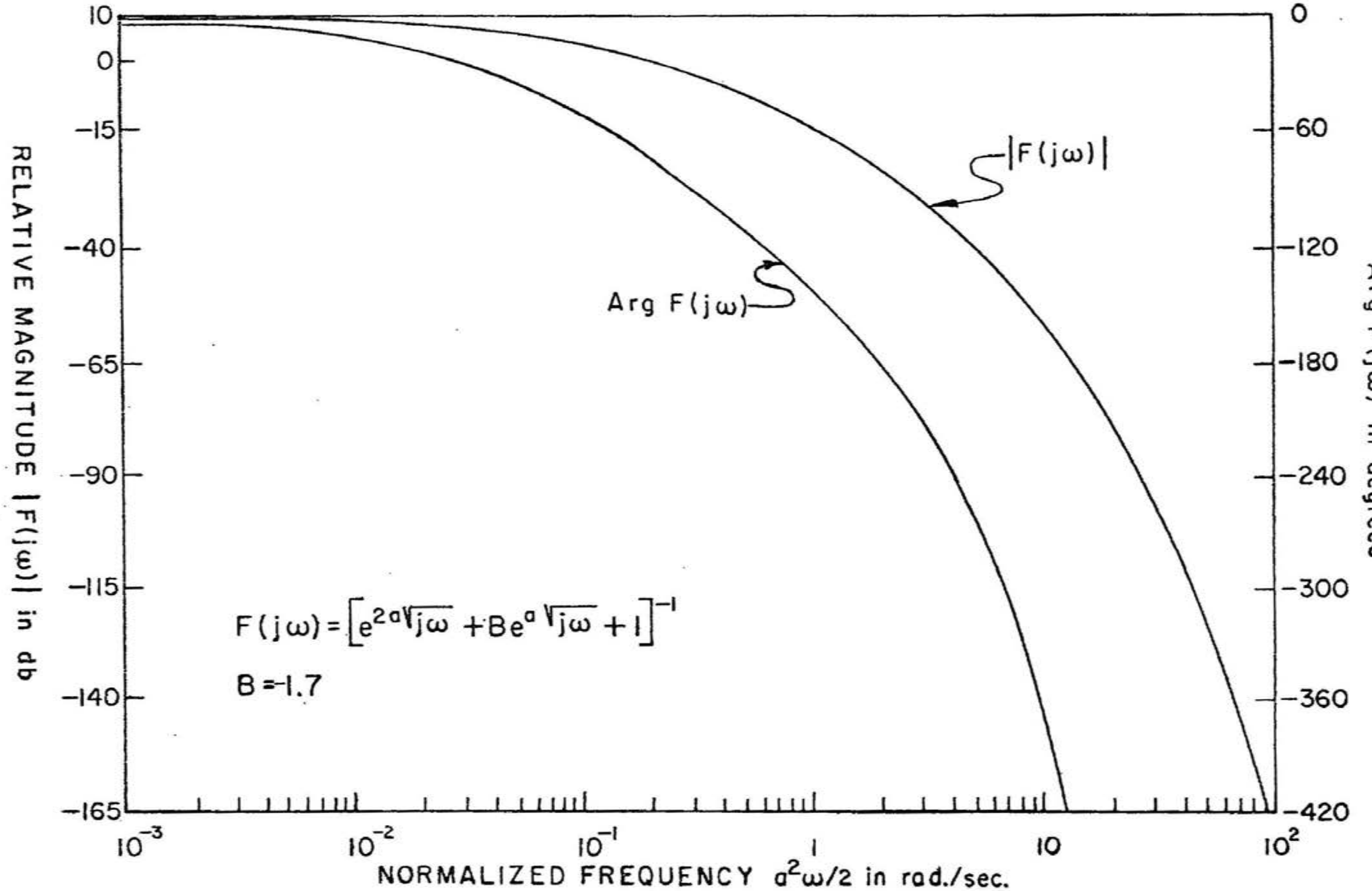

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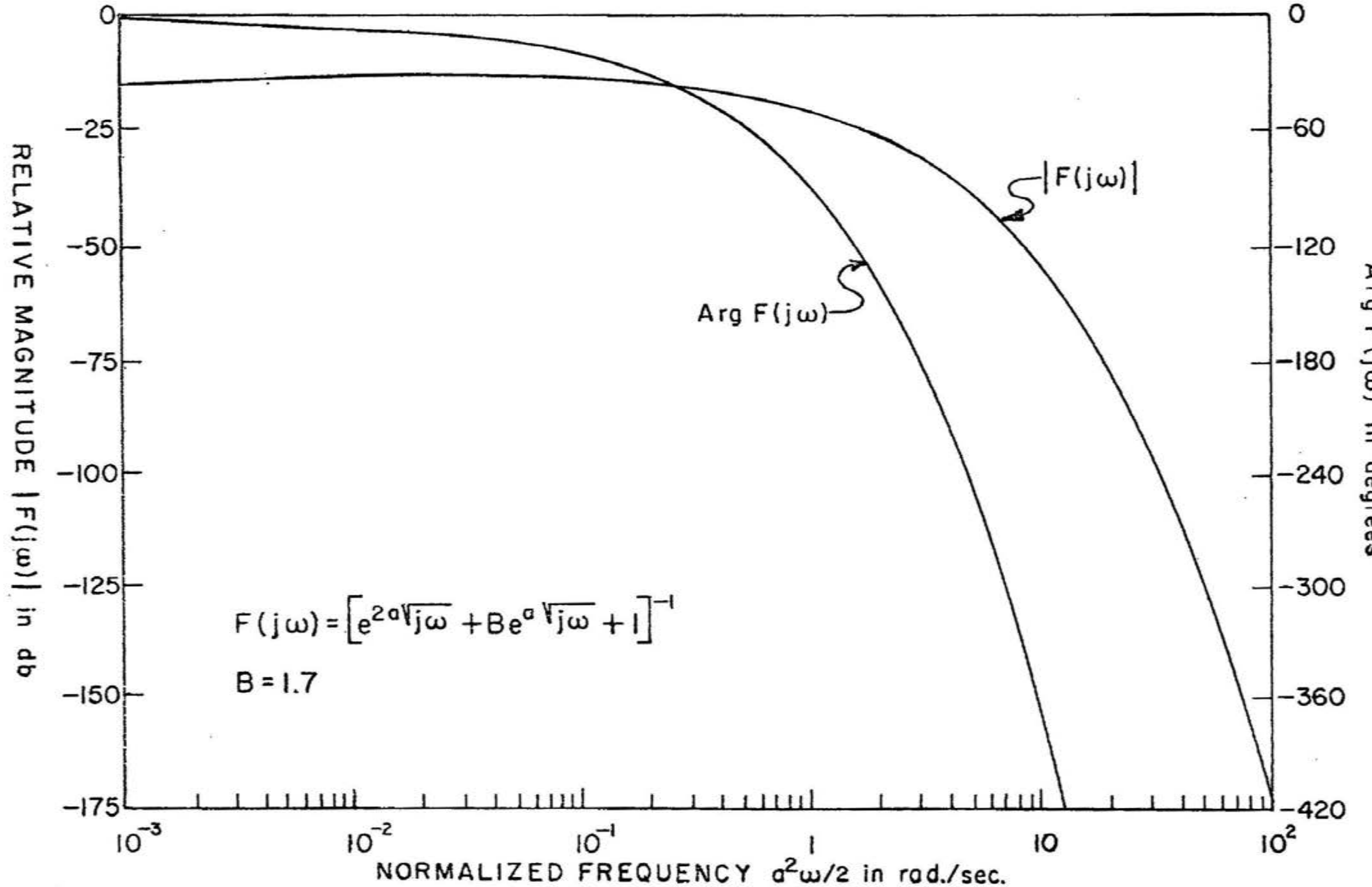

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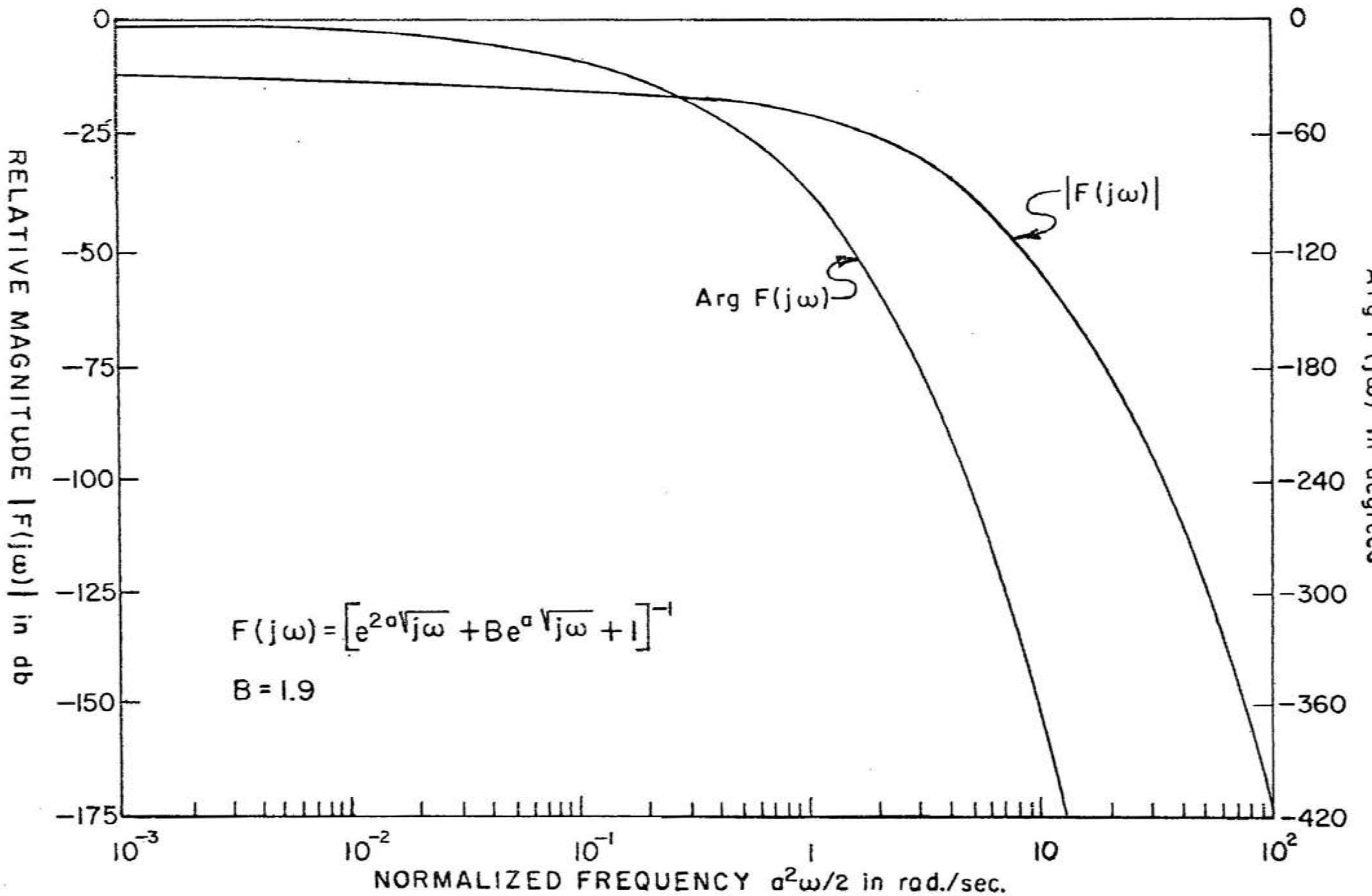

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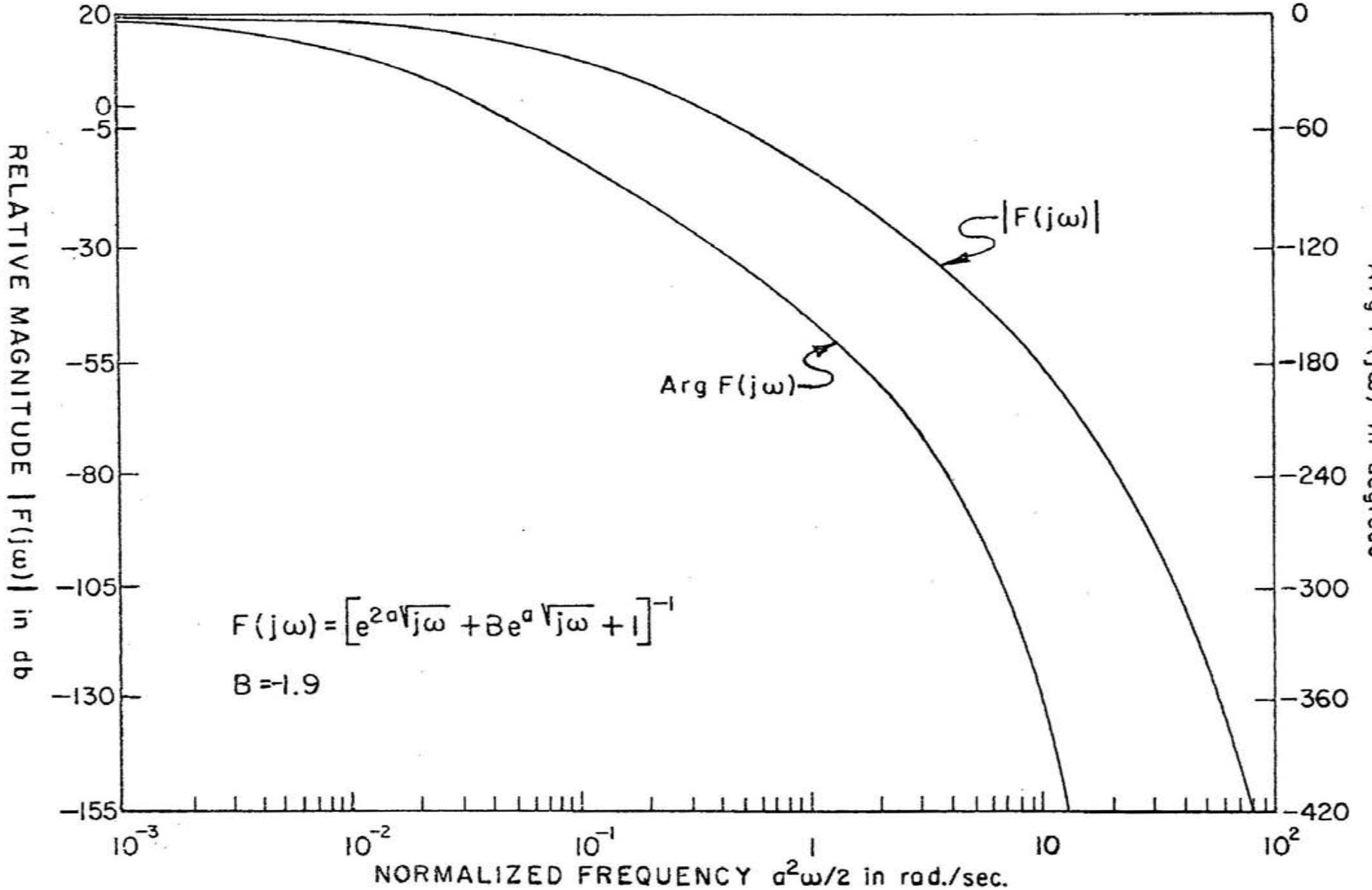

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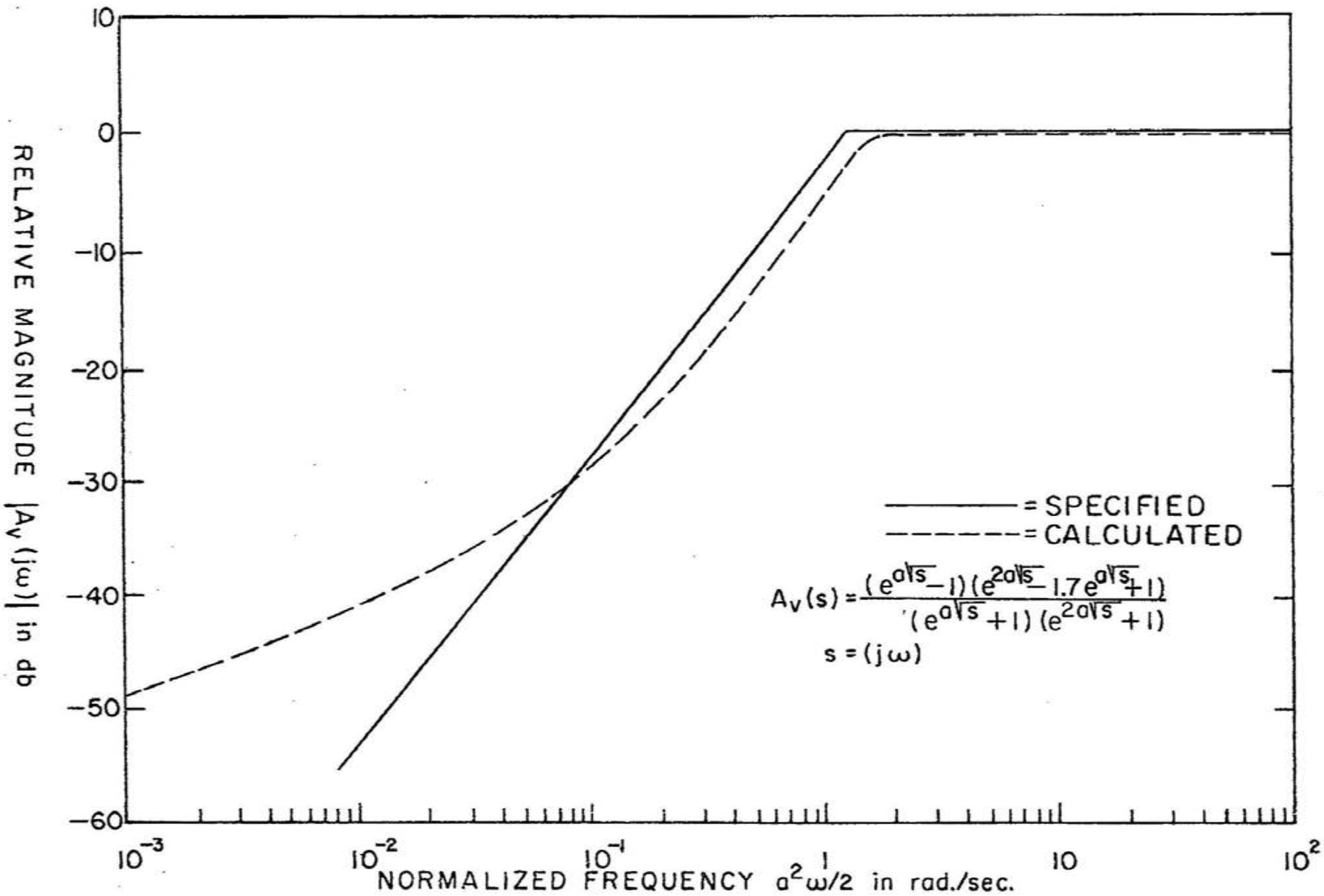

Figure No. 3-28 APPROXIMATION OF $A_v(s)$ BY EXPONENTIAL POLYNOMIALS

CHAPTER IV

THE SYNTHESIS OF LOW-PASS FILTERS

A. Preliminaries

What follows is due to Wyndrum[4] and others.[10,11]

1. Theorem I* If $Z_{RC}(s)$ is a driving point impedance of the distributed RC network $Z_{LC}(\sqrt{s}) = \sqrt{s}\, Z_{RC}(s)$ is a driving point impedance of the reactive network defined in the $\sqrt{s}$ plane.

Corollarly I If $Y_{RC}(s)$ is a driving point admittance of a distributed RC network, $Y_{LC}(\sqrt{s}) = \frac{1}{\sqrt{s}} Y_{RC}(s)$ is a driving point admittance of an LC network defined in the $\sqrt{s}$ plane. For the uniform distributed RC line segment,

$$z_{11} = \frac{R \coth \sqrt{sRC}}{\sqrt{sRC}} \tag{1}$$

$$\frac{1}{y_{11}} = \frac{R \tanh \sqrt{sRC}}{\sqrt{sRC}} \tag{2}$$

Under the above theorem and the corollarly the associated LC networks are characterized in the $\sqrt{s}$ plane by

$$z_{11_{LC}}(\sqrt{s}) = \sqrt{\frac{R}{C}} \coth \sqrt{s\,RC} \tag{3}$$

$$\frac{1}{y_{11_{LC}}(\sqrt{s})} = \sqrt{\frac{R}{C}} \tanh \sqrt{sRC} \tag{4}$$

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*For the proof of the theorem see the appendix of reference 4.

2. The "s $\rightarrow$ W" Transformation

Consider the positive real* mapping of figure 4-1:

$$W(\sqrt{s}) = \tanh \frac{a\sqrt{s}}{2} = \frac{\exp(a\sqrt{s}) - 1}{\exp(a\sqrt{s}) + 1} \tag{5a}$$

or

$$\exp(a\sqrt{s}) = \frac{1 + W}{1 - W} \tag{5b}$$

Under (5), (3) and (4) become

$$z_{11}(\sqrt{s}) = \sqrt{\frac{R}{C}}\,\frac{1}{W} \tag{6}$$

$$\frac{1}{y_{11}(\sqrt{s})} = \sqrt{\frac{R}{C}}\;W \tag{7}$$

The open and short circuited distributed RC segments in the s-plane are in a one to one correspondence with and may be replaced by inductors and capacitors respectively in the W-plane. Hyperbolic functions in the $\sqrt{s}$-plane are transformed into lumped LC immittance functions in the W-plane. Conventional LC synthesis techniques can be applied to realize one-port URCO and URCS element networks.

3. Richard's Theorem: Distributed RC Driving Point Synthesis

Richard's theorem[11] states: If Y(W) is a positive real function and

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*See page 357 of reference 10.

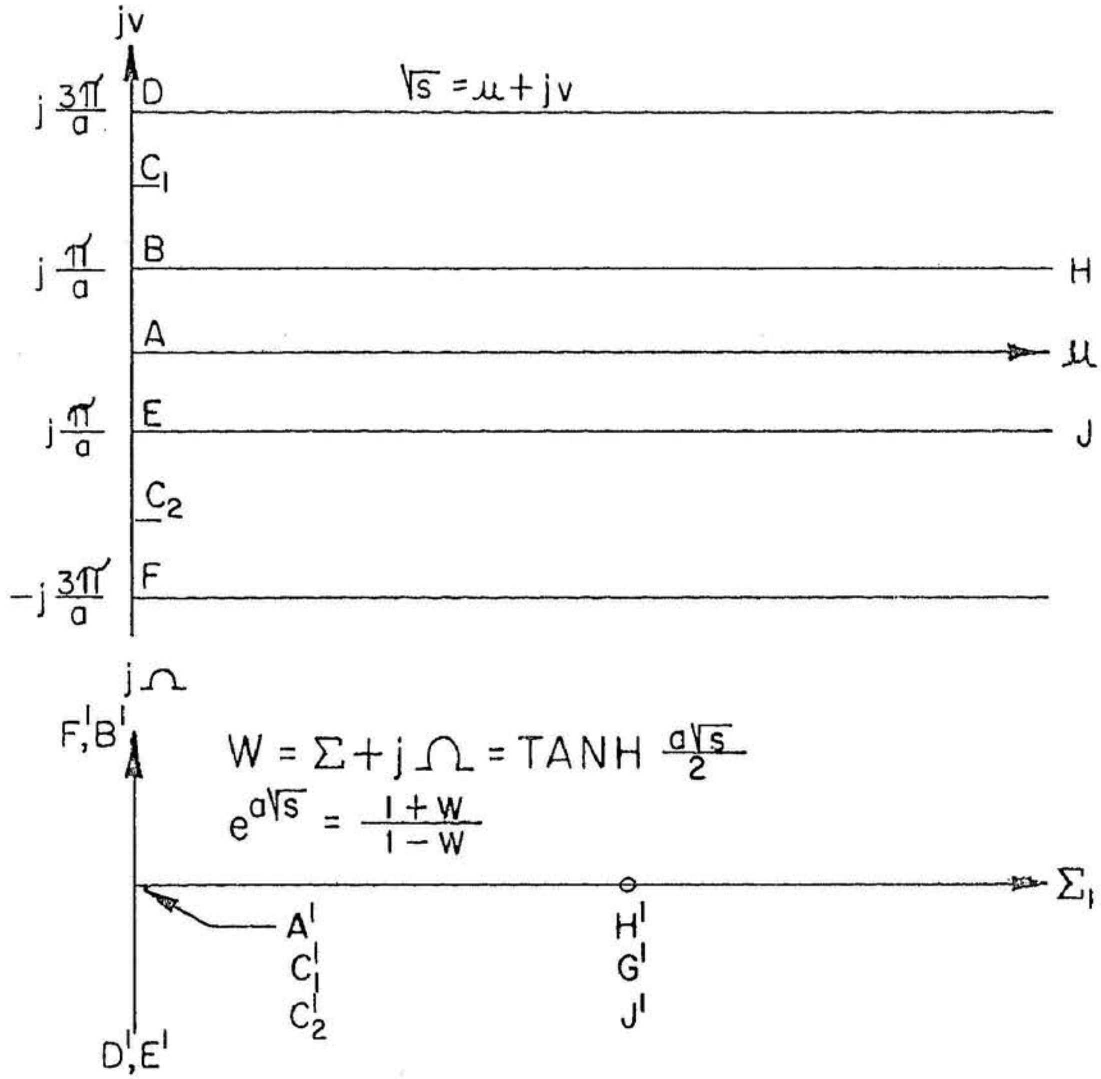


Figure No. 4-1 "S → W" TRANFORMATION

$$Y_1(W) = Y(1)\,\frac{Y(W) - W\,Y(1)}{Y(1) - W\,Y(W)} \tag{8}$$

is constructed, $Y_1(W)$ is again a positive real function. Additionally if $Y(W)$ is LC, $Y_1(W)$ must be LC.

In light of Richard's theorem consider the cascaded uniform distributed RC line segments (fig. 4-2) for which

$$Y_{i+1}(s) = \frac{1}{R_i}\;\frac{R_i Y_i(s) - \sqrt{s\tau}\,\tanh\sqrt{s\tau}}{1 - R_i Y_i(s)\,\dfrac{\tanh\sqrt{s\tau}}{\sqrt{s\tau}}} \tag{9}$$

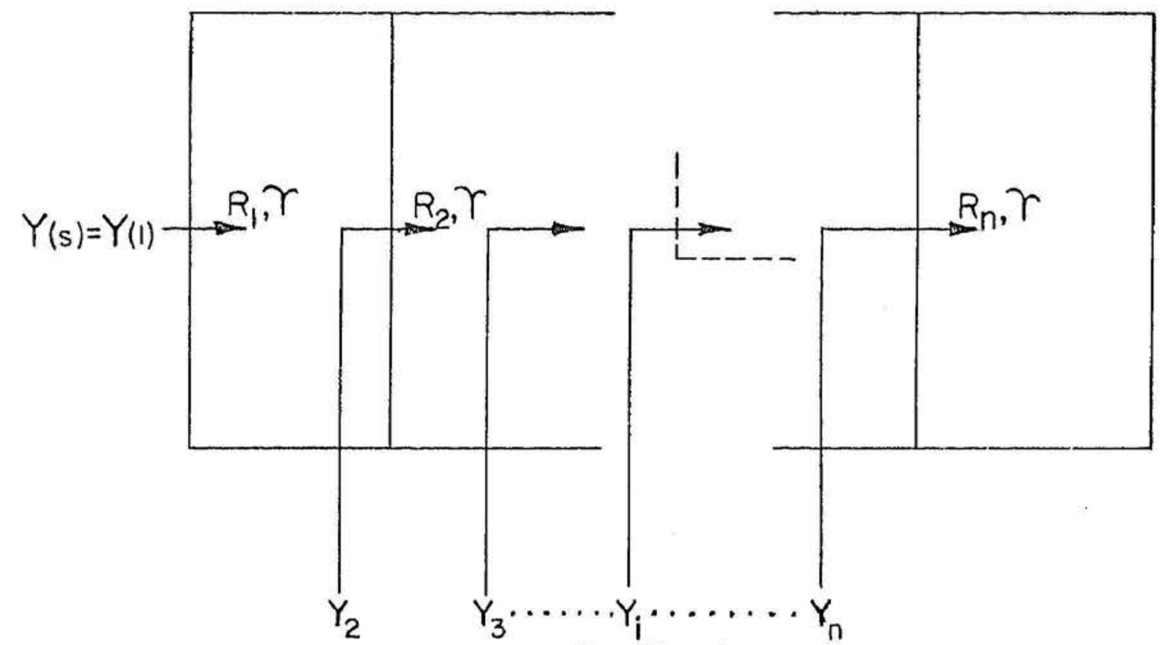
$$\tau = RC$$

where it is assumed that $Y_i(s)$ is a given and realizable distributed RC immittance function.

Under (5), in the W-plane

$$Y_{i+1}(W) = \frac{\sqrt{\tau}}{R_i}\;\frac{Y_i(W) - W\dfrac{\sqrt{\tau}}{R_i}}{\dfrac{\sqrt{\tau}}{R_i} - W\,Y_i(W)} \tag{10}$$

If $\frac{\sqrt{\tau}}{R_i}$ is chosen as $Y_i(1)$, equations (8) and (10) are identical and R_i is uniquely specified for each segment. The process is repeated until $Y_n(W)$ is equal to $\frac{1}{W}$ or W. The process will always converge.[11] The significance of the theorem is that distributed RC segments may always be extracted from a desired function, and the resulting simpler driving point admittance may always be realized as a cascade of a finite number of distributed RC segments through a repetition of the extraction process. The fact to be noted is that by Theorem I, (5), and the application of Richard's theorem (8), any distributed RC driving point function may be realized as a cascade of a finite number of distributed RC line



$$Y_{i+1}(s) = \frac{1}{R_i} \quad \frac{R_i Y_i(s) - \sqrt{s\Upsilon}\,\mathrm{Tanh}\sqrt{s\Upsilon}}{1 - R_i Y_i(s)\,\dfrac{\mathrm{Tanh}\sqrt{s\Upsilon}}{\sqrt{s\Upsilon}}}$$

Figure No. 4-2 CASCADED RC LINE SEGMENTS

segments. One of the advantages of this technique is the decreasing of the interconnections needed when one-port URCO and URCS networks are used.

B. Synthesis of Low-Pass Filters as a Cascade of Distributed RC Segments

If a low-pass filter characteristic is required to have the sharpest cut-off practically possible for a given number of segments with a monotonically decreasing function, its transfer function will lack any second order exponential polynomial in the numerator.*

According to Wyndrum[4] the general two-port admittance functions for low-pass filters are:

$$- y_{21}(s) = \frac{\sqrt{s}\,(\exp(a\sqrt{s}) + 1)^{2m-q+1} \exp(qa\sqrt{s}/2)}{(\exp(a\sqrt{s}) - 1)\prod_{k=1}^{m}(\exp(2a\sqrt{s}) + A_k \exp(a\sqrt{s}) + 1)} \tag{11}$$

and

$$y_{22}(s) = \frac{\sqrt{s}\,(\exp(a\sqrt{s}) + 1)^{2m-q+1} \prod_{1}^{n}(\exp(2a\sqrt{s}) + B_i \exp(a\sqrt{s}) + 1)}{(\exp(a\sqrt{s}) - 1) \prod_{1}^{m}(\exp(2a\sqrt{s}) + A_k \times \exp(a\sqrt{s}) + 1)} \tag{12}$$

where

"q' is the number of cascaded segments.

$m = q/2 - 1$, $n = m$, or $m + 1$, $|A_k| < 2$, $|B_i| < 2$, $A_k > B_i$

then

$$A_v(s) = \frac{-y_{21}(s)}{y_{22}(s)} = \frac{\exp(pa\sqrt{s}/2)}{\prod_{1}^{n}(\exp(2a\sqrt{s}) + B_i \times \exp(a\sqrt{s}) + 1)} \tag{13}$$

---

*See reference 4, pages 103-104.

The sharpness with which the frequency characteristic of $A_v(s)$ breaks depends on the number of B_i terms which may be close to -2. The values of A_k can be chosen by knowing the fact that $A_k > B_i$ and that because of the pole-zero alternation in the W-plane A_k and B_i must alternate.[4]

A synthesis example of the eighth order, frequency and magnitude normalized, low-pass filter follows. The calculated data is given in fig. 4-3. The filter was not designed to meet specific requirements, but rather to find how sharp a cutoff might be realized by an eight-section distributed RC filter, and how the phase characteristic would be. From (12),

$$y_{22}(s) = \frac{\sqrt{s}\,\prod_1^4 \left(\exp(2a\sqrt{s}) + B_i\exp(a\sqrt{s}) + 1\right)}{\left(\exp(a\sqrt{s}) + 1\right)\left(\exp(a\sqrt{s}) - 1\right)\prod_1^3 \left(\exp(2a\sqrt{s}) + A_k\exp(a\sqrt{s}) + 1\right)} \tag{14}$$

Application of the "s → W" transformation yields

$$Y_{22}(W) = \frac{\prod_1^4\left[\left(\frac{1+W}{1-W}\right)^2 + B_i\left(\frac{1+W}{1-W}\right) + 1\right]}{\left[\left(\frac{1+W}{1-W}\right)^2 - 1\right]\prod_1^3\left[\left(\frac{1+W}{1-W}\right)^2 + A_k\left(\frac{1+W}{1-W}\right) + 1\right]} \tag{15}$$

$$= \frac{K\left(W^2 + \frac{2+B_1}{2-B_1}\right)\left(W^2 + \frac{2+B_2}{2-B_2}\right)\left(W^2 + \frac{2+B_3}{2-B_3}\right)\left(W^2 + \frac{2+B_4}{2-B_4}\right)}{W\left(W^2 + \frac{2+A_1}{2-A_1}\right)\left(W^2 + \frac{2+A_2}{2-A_2}\right)\left(W^2 + \frac{2+A_3}{2-A_3}\right)}$$

where

$$K = \frac{(2-B_1)\,(2-B_2)\,(2-B_3)\,(2-B_4)}{4\,(2-A_1)\,(2-A_2)\,(2-A_3)}$$

The B_i are arbitrarily selected (close to "-2").

$$B_1 = -1.9,\ B_2 = -1.7,\ B_3 = -1.5,\ B_4 = -1.3$$

the A_k are then chosen so that they alternate with the B_i:

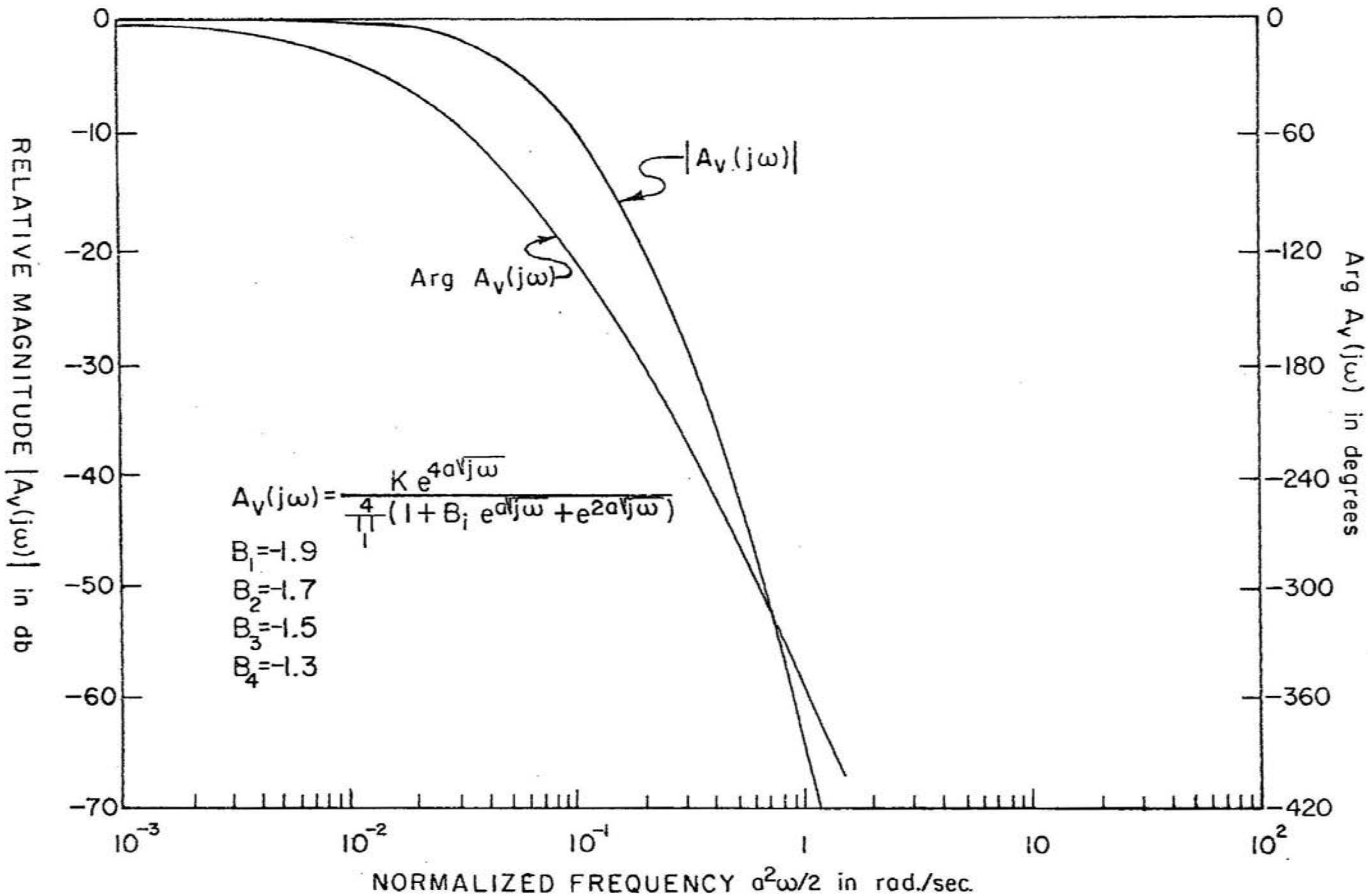


Figure No. 4-3 EIGHT-SECTION LOW-PASS FILTER CHARACTERISTICS

$$A_1 = -1.8,\ A_2 = -1.6,\ A_3 = -1.4$$

$$k = 0.896$$

$$y22(W) = 0.896\,\frac{(W^2 + 1/39)(W^2 + 3/37)(W^2 + 5/35)(W^2 + 7/33)}{W(W^2 + 2/38)(W^2 + 4/36)(W^2 + 6/34)}$$

$$y_{22}(W) = y(W)$$

$$y(1) = 1.0 \text{ (from eq. (15))}$$

Application of Richard's Theorem gives

$$y_1(W) = y(1)\,\frac{y(W) - W y(1)}{y(1) - W y(W)}$$

$$= \frac{10.42\,W^6 + 3.08W^4 + 0.26\,W^2 + 0.01}{W\,(89.58\,W^6 + 30.94\,W^4 + 3.21\,W^2 + 0.1)}$$

$$y_1\,(1) = 0.111$$

$$y_2\,(W) = \frac{11.06\,W^6 + 3.30\,W^4 + 0.28\,W^2 + 0.01}{W\,(4.64\,W^4 + 1.04\,W^2 + 0.05)}$$

$$y_2\,(1) = 2.557$$

$$y_3\,(W) = \frac{20.37\,W^4 + 3.98\,W^2 + 0.16}{W(110.60\,W^4 + 25.08\,W^2 + 1.25)}$$

$$y_3\,(1) = 0.179$$

$$y_4\,(W) = \frac{35.51\,W^4 + 7.05\,W^2 + 0.312}{W(5.59\,W^2 + 0.508)}$$

$$y_4\,(1) = 7.027$$

$$y_5(W) = \frac{26.64\,W^2 + 2.19}{W(35.51\,W^2 + 3.26)}$$

$$y_5(1) = 0.743$$

$$y_6(W) = \frac{19.63\,W^2 + 1.63}{0.234\,W}$$

$$y_6(1) = 90.925$$

$$y_7(W) = \frac{1.480}{0.196\,W}$$

$$y_7(1) = 7.540$$

In summary,

$$y(1) = \frac{\sqrt{\tau}}{Ro} = 1.0$$

$$y_1(1) = \frac{\sqrt{\tau}}{R_1} = 0.111$$

$$y_2(1) = \frac{\sqrt{\tau}}{R_2} = 2.556$$

$$y_3(1) = \frac{\sqrt{\tau}}{R_3} = 0.179$$

$$y_4(1) = \frac{\sqrt{\tau}}{R_4} = 7.027$$

$$y_5(1) = \frac{\sqrt{\tau}}{R_5} = 0.743$$

$$y_6(1) = \frac{\sqrt{\tau}}{R_6} = 90.92$$

$$y_7(1) = \frac{\sqrt{\tau}}{R_7} = 7.540$$

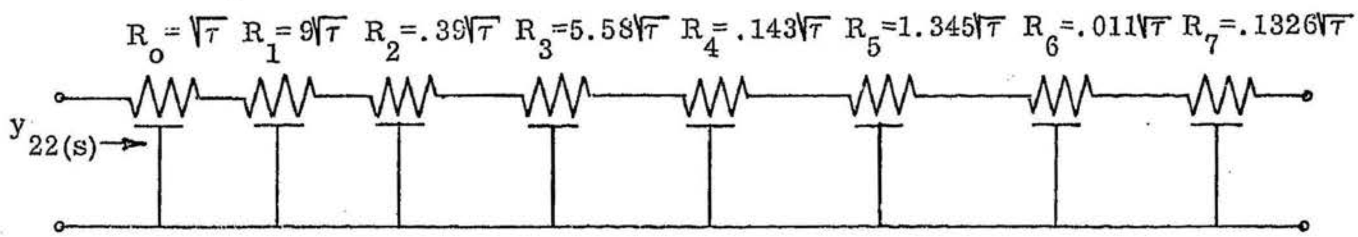


FIG 4-4
DISTRIBUTED RC LOW-PASS FILTER

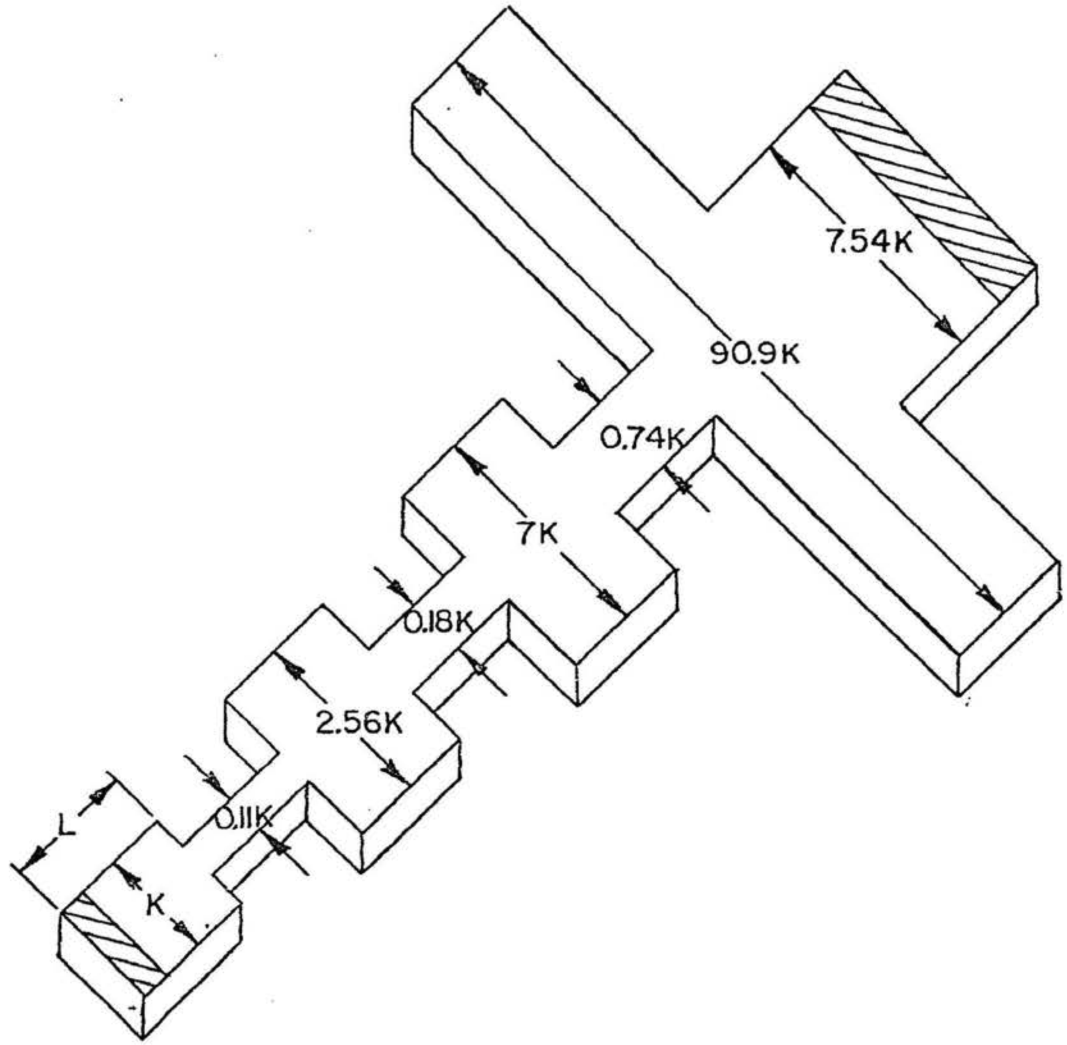


Figure No. 4-5 LOW-PASS FILTER

$\sqrt{\tau}$ may be chosen for the proper frequency scaling. The physical embodiment of the normalized network appears in fig. 4-5.

C. Symmetrical Lattice of Distributed RC Segments

The magnitude, phase, and impedance characteristic of a delay network realized as a symmetrical lattice composed of one-port URCO and URCS networks (fig. 4-6) are shown in figs. 4-7 and 4-8. As in the cascade problem $\sqrt{\tau}$ may be chosen for the proper frequency scaling.

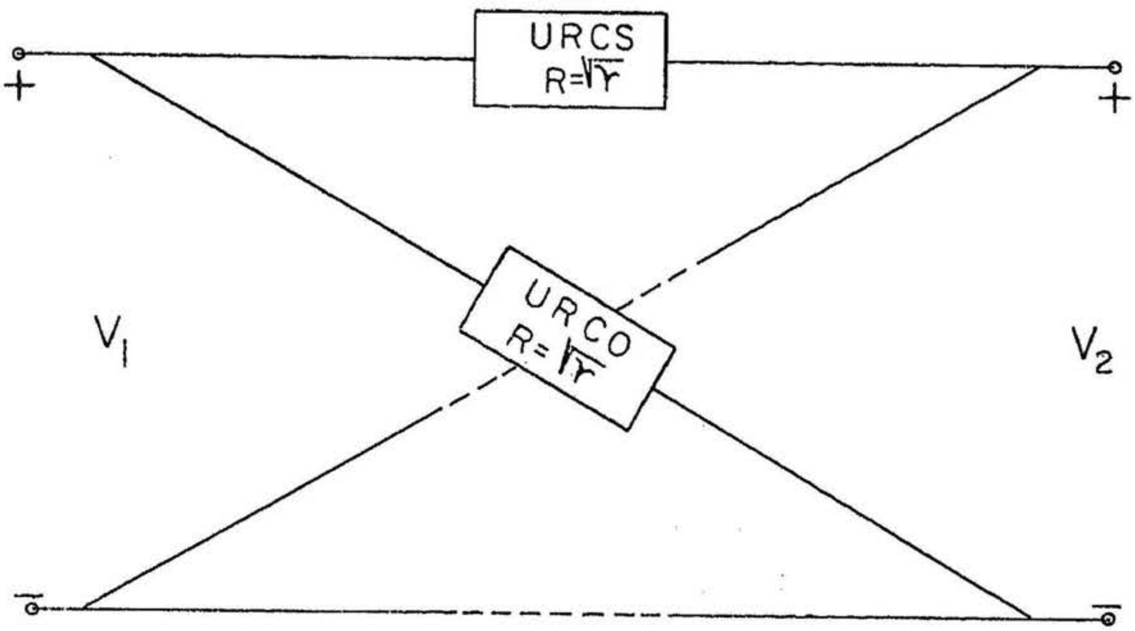


FIG 4-6
DISTRIBUTED RC DELAY NETWORK

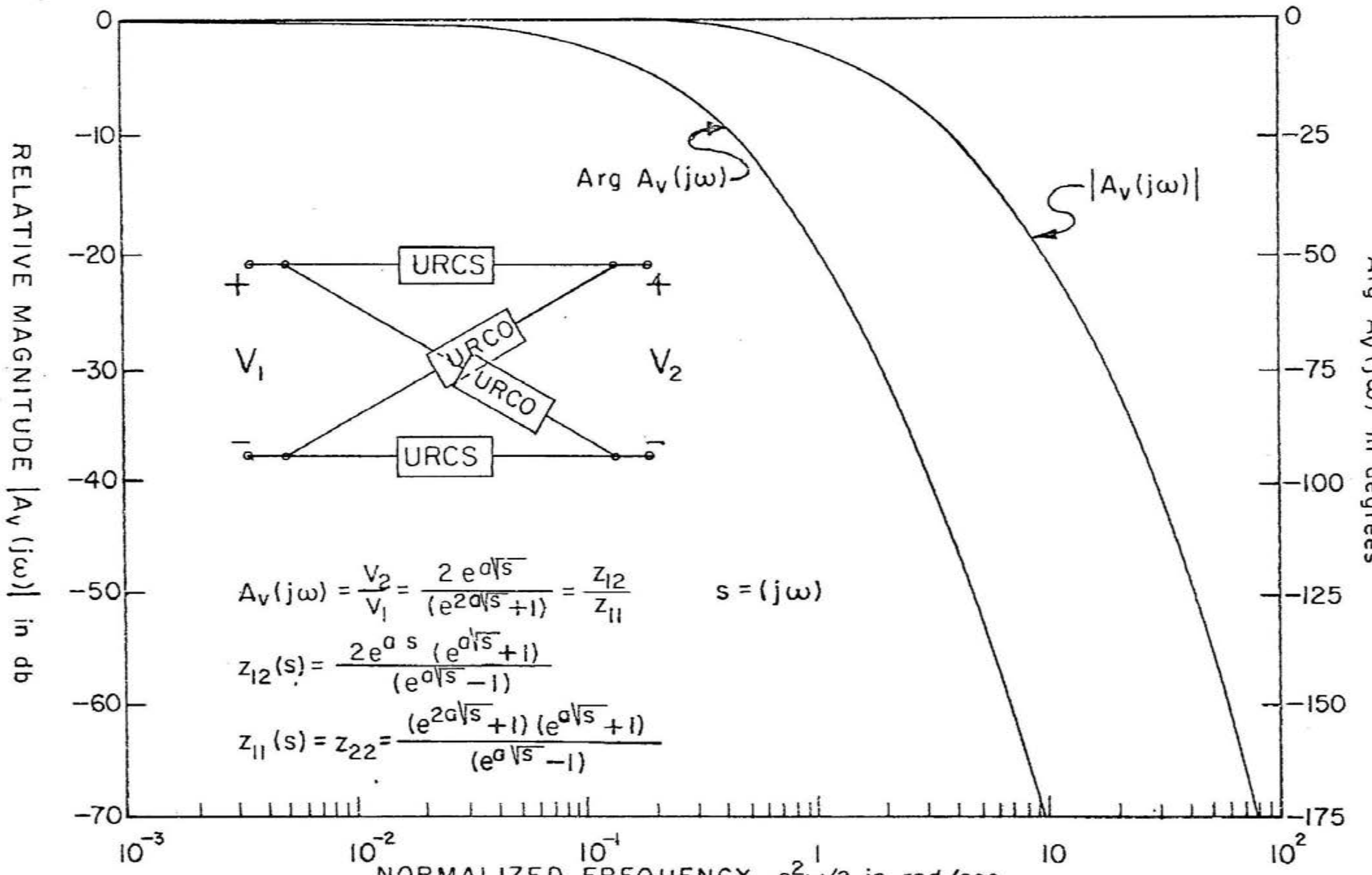


Figure No. 4-7 DELAY NETWORK CHARACTERISTICS

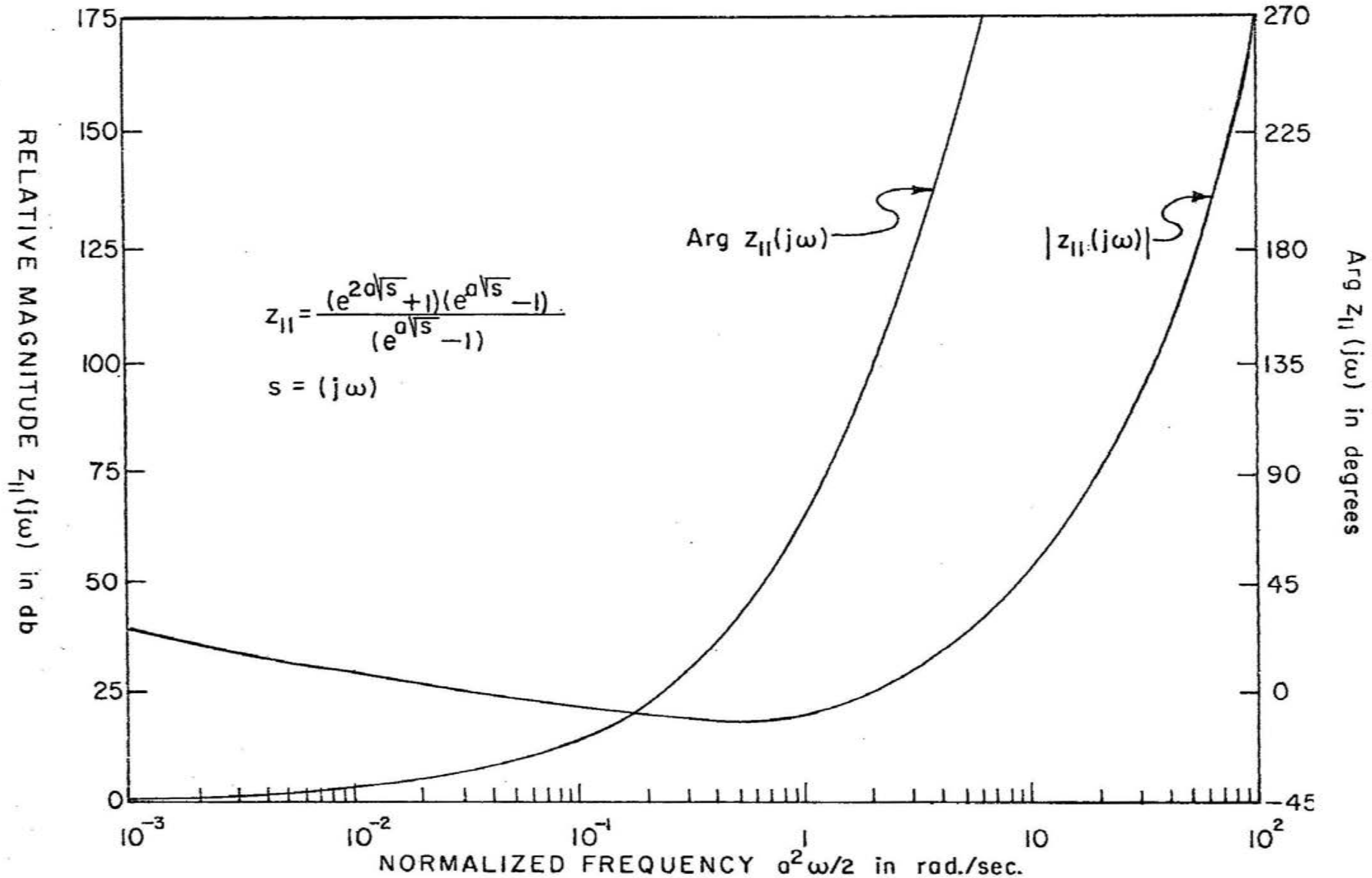


Figure No. 4-8 DELAY NETWORK CHARACTERISTIC IMPEDANCE

CHAPTER V

SYNTHESIS OF A HIGH-PASS FILTER

The specifications for this problem are:

A gain of zero for zero frequency

A gain of one for the normalized frequency $\frac{a^2\omega}{2} \geq 1$

An attenuation of at least 40 db for $\frac{a^2\omega}{2} \leq 0.1$

Because all the zeros of transmission for this filter are at zero frequency, the term " $(\exp(a\sqrt{s}) - 1)$" is needed in the numerator of the transfer function. The denominator factors are chosen to satisfy the other specifications. From the plots of Chapter III a satisfactory transfer function was found to be:

$$A_v(s) = \frac{(\exp(a\sqrt{s}) - 1)^6}{\prod_1^3 (\exp(2a\sqrt{s}) + B_i \exp(a\sqrt{s}) + 1)} \tag{1}$$

where

$$B_1 = -1.1, \qquad B_2 = -0.7, \qquad B_3 = 1.2$$

The magnitude and phase characteristics for the normalized frequency "$a^2\omega/2$" are shown in fig. 5-1.

This filter is going to be synthesized as a ladder of one-port URCO and URCS networks.

Applying the "$s \to W$" transformation to (1)

$$A_v(W) = \frac{W^6}{(W^2 + 0.291)(W^2 + 0.482)(W^2 + 4.0)} = \frac{-y_{21}(W)}{y_{22}(W)}$$

The denominator of $y_{21}(W)$ and $y_{22}(W)$ is chosen according to the realizability conditions for lumped LC two-port networks.[12]

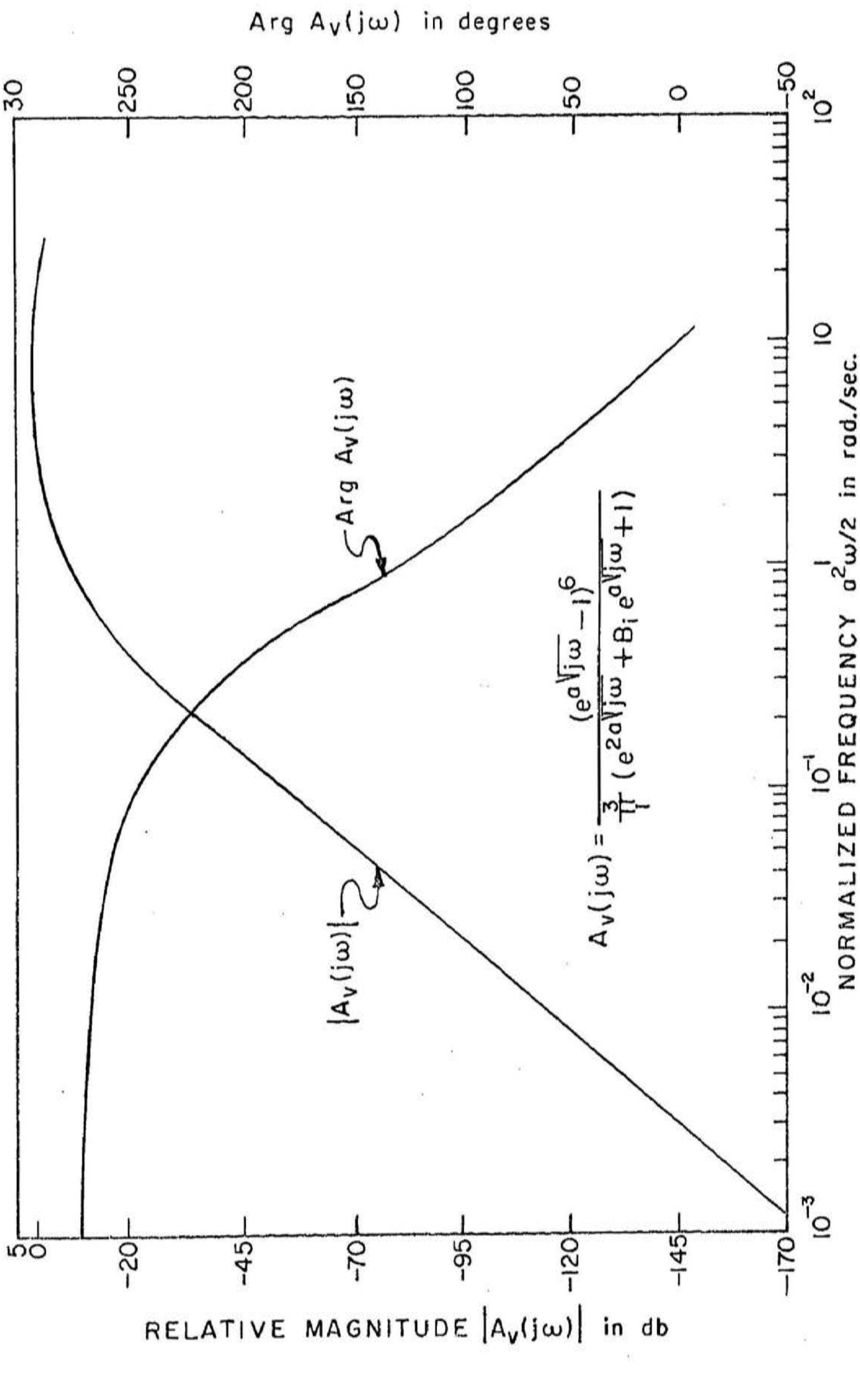


Figure No. 5-1 HIGH-PASS FILTER CHARACTERISTICS

$$-y_{21}(W) = \frac{W^7}{(W^2 + 1)(W^2 + 0.333)(W^2 + 0.25)}$$

$$y_{22}(W) = \frac{W(W^2 + 0.291)(W^2 + 0.482)(W^2 + 4.0)}{(W^2 + 1)(W^2 + 0.333)(W^2 + 0.25)}$$

$y_{22}(W)$ is realized using the ladder development as is $-y_{21}(W)$ (within a constant).

The ladder of lumped elements is shown in fig. 5-2

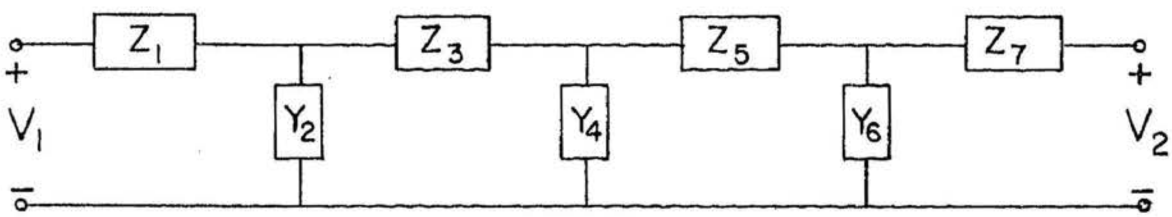


FIG 5-2
LADDER LUMPED LC NETWORK

where

$$Z_1 = \frac{1}{6.14W} \qquad Y_2 = \frac{1}{0.32W} \qquad Z_3 = \frac{1}{2.61W}$$

$$Y_4 = \frac{1}{0.33W} \qquad Z_5 = \frac{1}{3.26W} \qquad Y_6 = \frac{1}{0.333W}$$

$$Z_7 = \frac{1}{6.72W}$$

Applying eqs. (6) and (7) of Chapter IV it is possible to determine the parameters for the distributed network. This network is shown in fig. 5-3. One practical disadvantage of this network is that it has too many interconnections.

$\sqrt{T}$ is chosen for the proper frequency scaling.

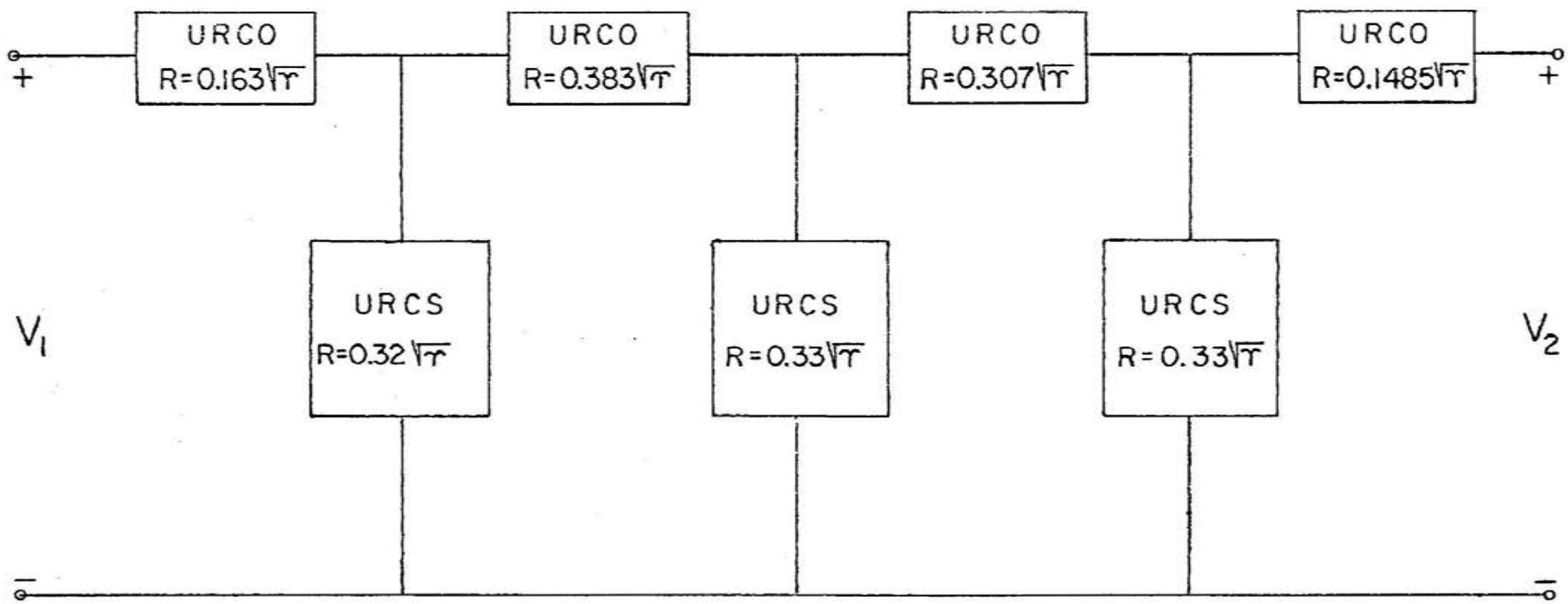


Figure No. 5-3 DISTRIBUTED RC HIGH-PASS FILTER

CHAPTER VI

SYNTHESIS OF A BAND-PASS FILTER

A. Preliminaries*

Transmission zeros in the W and the s-planes: in the W-plane, transmission zeros may generally be realized anywhere except for the region of the positive real axis between the origin and unity. This segment corresponds to the entire positive real axis in the s-plane. Generally, then zeros may exist with quadrantal symmetry in the W-plane, or in conjugate symmetry on the $j\Omega$ axis. In the W-plane they will be finite in number. Conjugate imaginary zeros in the W-plane map into the negative purely real zeros in the s-plane.

It can be shown that because of the nature of the cascaded distributed RC elements, finite transmission zeros on the imaginary W axis cannot be realized using these elements only. In order to realize them, stubs (which are really shunt elements) have to be employed to modify the cascade type synthesis used in Chapter IV. This technique is called A "cascade and stub" synthesis procedure. Figures 6-1 and 6-2 illustrate the new configuration and the procedure for the synthesis in the W-plane respectively.

A "resonant stub" in the W-plane represents a shunt element which realizes one pair of conjugate imaginary W-plane zeros of transmission by effecting a "short circuit" at the frequency of interest. This is like removing one term of the form "($\exp(2a\sqrt{s}) + B\exp(a\sqrt{s}) + 1$)" from the corresponding

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*For the explanation of the statements of this part see references 4 and 10.

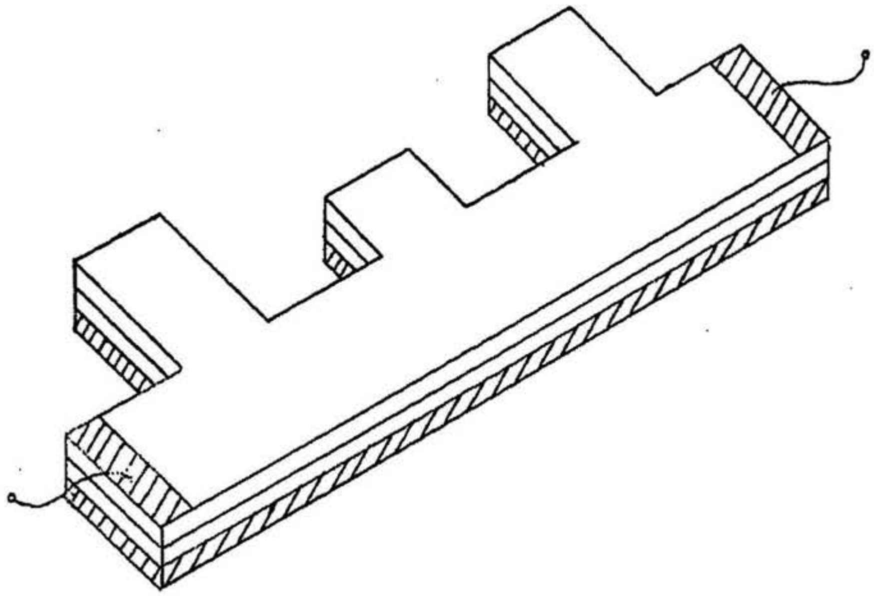

Figure No. 6-1 A CASCADE-STUB REALIZATION

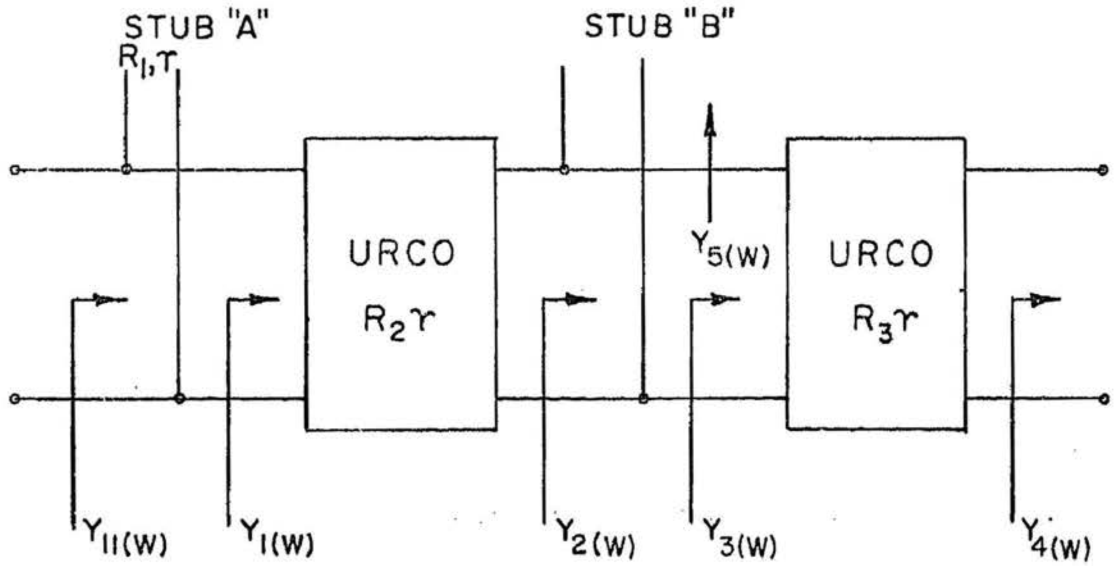

Figure No. 6-2 CASCADE-STUB EXTRACTION CYCLE s-plane expression. Other RC elements are then extracted to complete the cycle associated with the desired transmission zeros, following the approach of the work of Ikeno[13] in his lossless coaxial filter design.

Wyndrum[4] discussed the transfer function extraction cycle for finite zeros. The specific procedure to synthesize $y_{22}(W) = Y(W)$ and $y_{21}(W)$ simultaneously follows, $y_{22}(W) = Y(W)$ is such that either

$$Y(\infty) = 0 \text{ or } Y(\infty) = \infty$$

If the former is true, a cascade section may always be extracted, using Richard' s theorem so that

$$Y(\infty) = \infty \tag{1}$$

When (1) is satisfied, an URCO element may always be removed whose minimum total resistance is given by

$$R_o = \sqrt{\tau}W/\left. Y(W) \right|_{W \to \infty} \tag{2a}$$

and whose total capacitance is given by

$$C_o = \sqrt{\tau}\left. Y(W)/W \right|_{W \to \infty} \tag{2b}$$

At a finite transmission zero jQ_i in the W-plane, $Y(jQ_i)$ is purely imaginary. For each zero of transmission compute

$$R_{oi} = (1 + Q^2)/(Y(1)-jQY(jQ)) \tag{3}$$

Of the set $< R_{oi}, R_o >$, some elements may be negative, some will be positive. Remove the largest positive member, R_1, as a shunt URCO stub of admittance $W\sqrt{\tau}/R_1$. The remaining admittance is seen to be

$$Y_1(W) = Y(W) - W\sqrt{\tau}/R_1 \tag{4}$$

Since

$$\sqrt{\tau}/R_1 < \sqrt{\tau}/R_o$$

$$Y_1(\infty) = \infty \tag{5}$$

Remove a cascade element, $Y_1(1)$, from $Y_1(W)$, and by Richard's theorem the remaining admittance is given by

$$Y_2(W) = Y_1(1)\,\frac{Y_1(W) - W\,Y_1(1)}{Y_1(1) - W\,Y_1(W)} \tag{6}$$

Now it is seen that $Y_2(W)$ has a pole wherever $Y_1(1) = W\,Y_1(W)$. But at a transmission zero, $Y_1(1) = jQY_1(jQ)$.* Hence $Y_2(W)$ must have a pole at the required transmission zero. This pole may be removed by a stub whose admittance is given by

$$Y_5(W) = \frac{W\left[Y_2(W)\,\frac{W^2 + Q^2}{W}\right]_{W \to jQ}}{W^2 + Q^2} \tag{7}$$

which may be realized as a series of an URCO and URCS.

The remaining admittance $Y_3(W)$ is

$$Y_3(W) = Y_2(W) - Y_5(W) \tag{8}$$

Thus $Y_3(\infty) = 0$. A cascade element may again be removed and $Y_4(\infty) = \infty$. The cycle is complete.

At this point it is important to state the corollary of a theorem concerning the realizability of transmission zeros in the W-plane. This is due to Kasahara and Fujisawa, and reported by Ikeno.[13]

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*See reference 4 page 74.

Corollary II: The zeros of transmission of y_{21} are realizable simultaneously with a given driving point admittance y_{22} using no negative elements if

a. $W = \infty$ is a transmission zero of $y_{21}(W)/W2$.

b. All finite $j\Omega$ $(W = \Sigma + j\Omega)$ transmission zeros of y_{21} are greater than or equal to the largest pole of $y_{21}(W)$.

c. At least one finite $j\Omega$ transmission zero is greater than or equal to the greatest finite zero of $y_{22}(W)$.

B. Band-pass Filter Synthesis Example

Figs. 6-3 and 6-4 show the required configuration and the required gain characteristic for the band-pass filter.

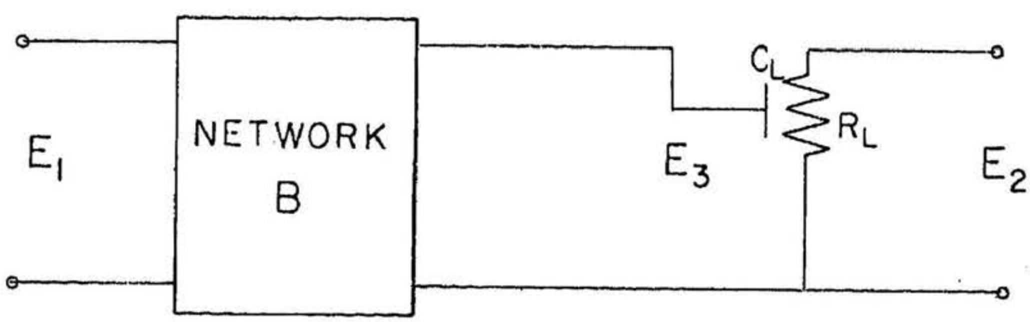

FIG 6-3
REQUIRED TRANSFER NETWORK CONFIGURATION

It will be assumed that the RC product of the uniform distributed RC segments of network B will be the same as the $R_L C_L$ product. The characteristic of the load network, E_2/E_3, is known and presented in fig. 6-5. To find the voltage gain characteristic E_3/E_1 for the network B, loaded by the output network, the characteristic of fig. 6-5 may be subtracted from that of fig. 6-4 (from the exact plot, not the Bode approximation). The resulting gain function

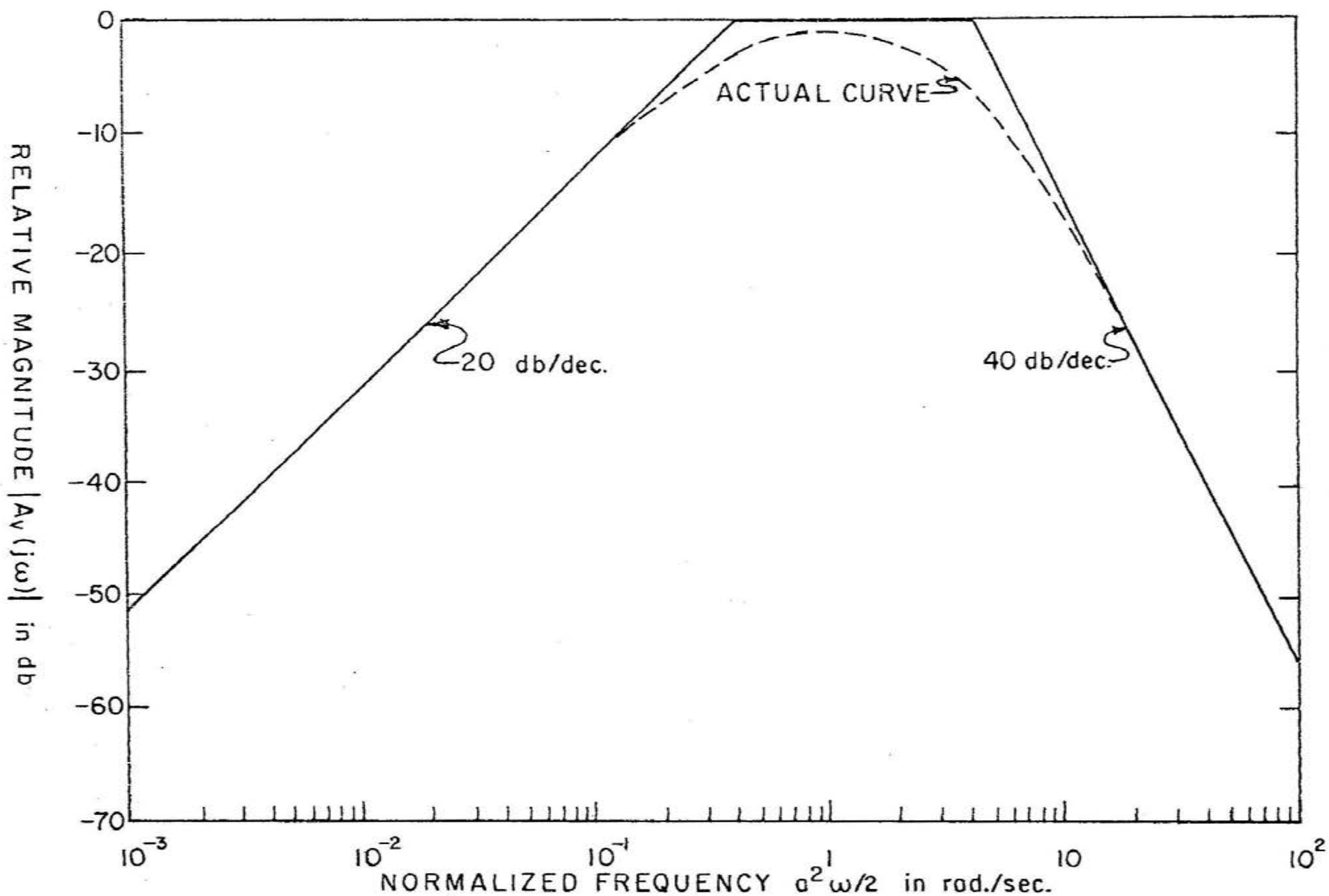

Figure No. 6-4 REQUIRED GAIN CHARACTERISTIC

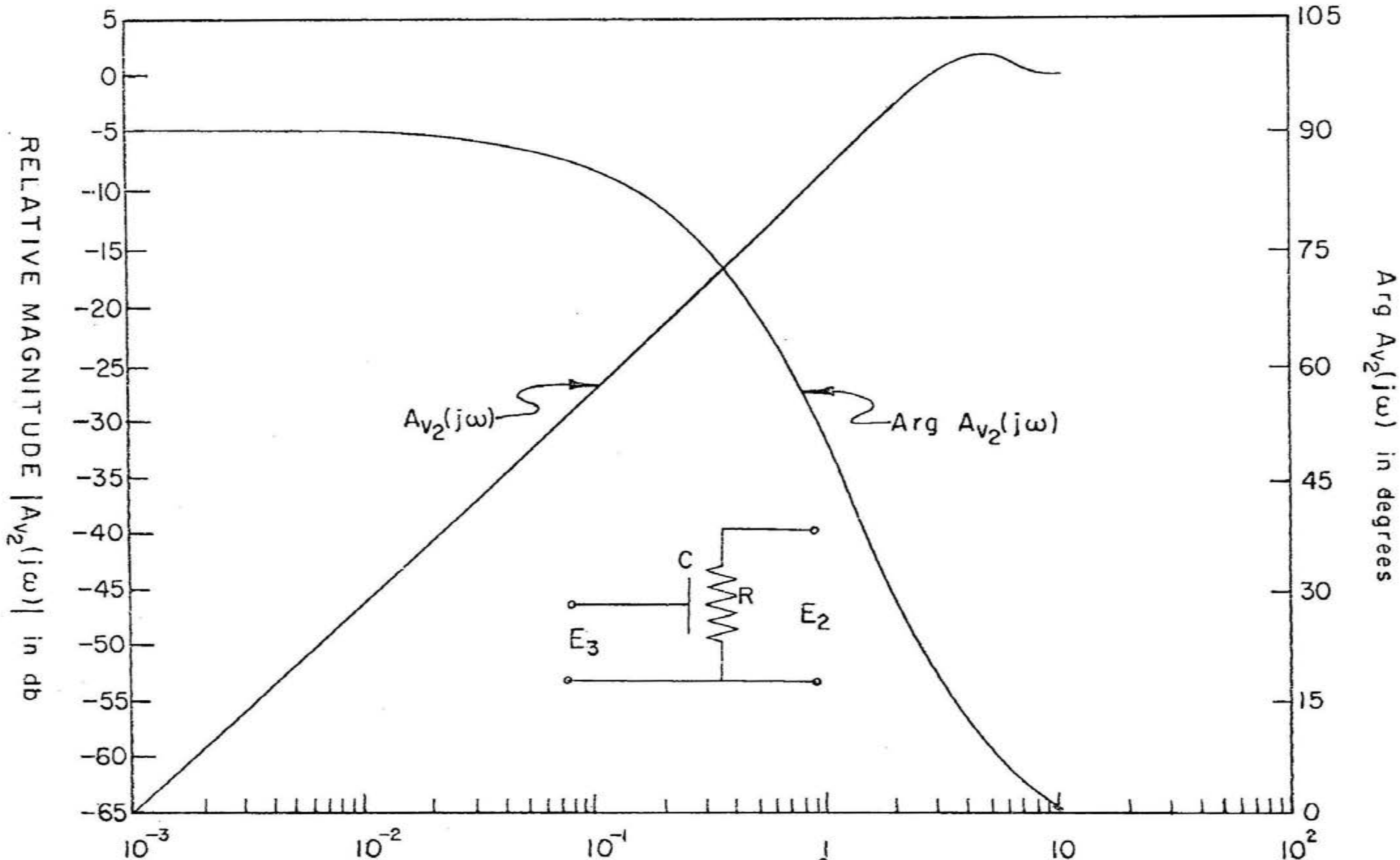

Figure No. 6-5 DISTRIBUTED RC NETWORK GAIN AND PHASE CHARACTERISTIC

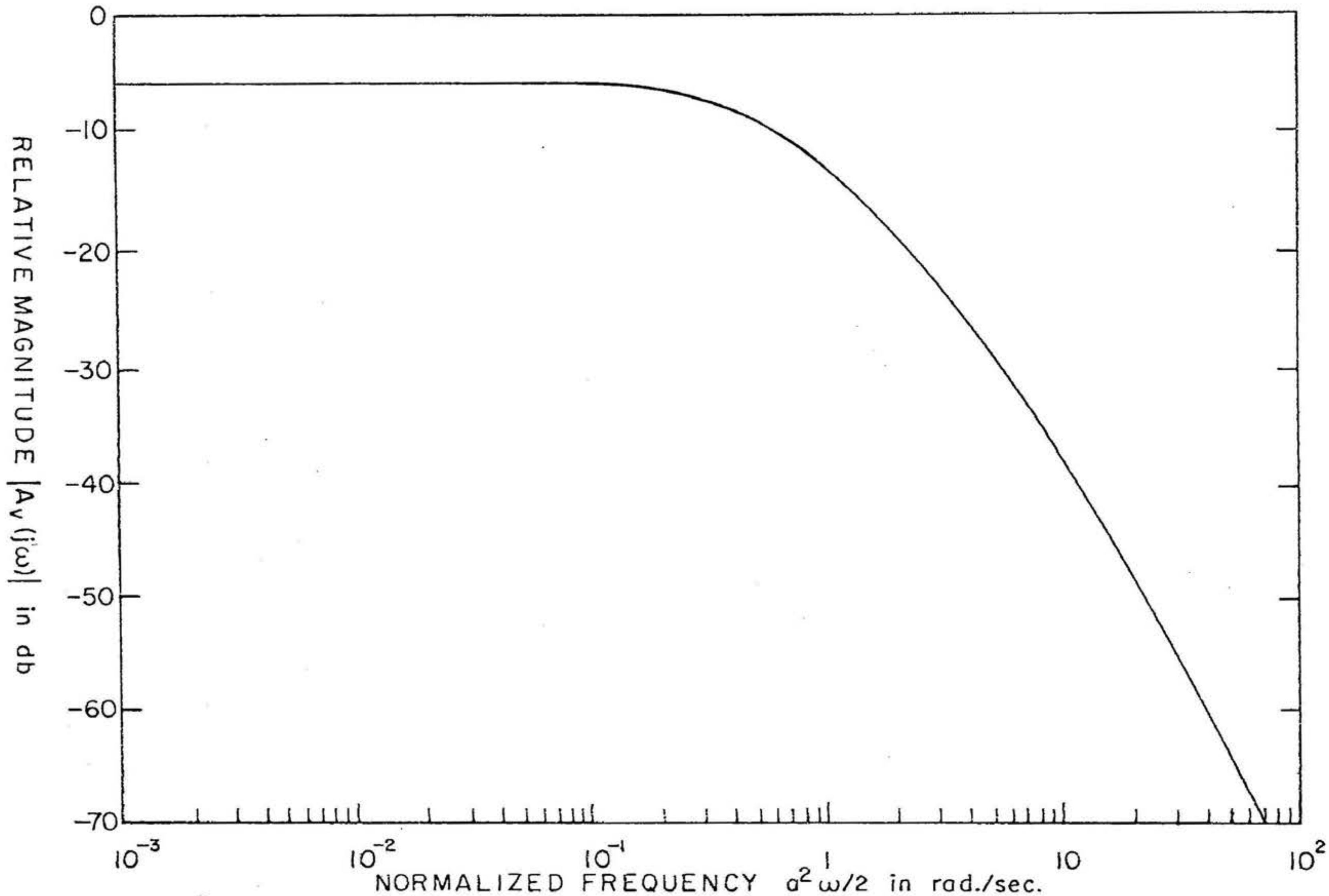

Figure No. 6-6 REQUIRED GAIN CHARACTERISTIC FOR NETWORK "B".

for B is plotted in fig. 6-6. Network B will be designed to meet the graphical specification of fig. 6-6, within an arbitrary constant.

Applying the techniques of Chapter III, the characteristic for the network B may be approximated closely by

$$A_v(s) = \frac{\exp(a\sqrt{s})\ (\exp(2a\sqrt{s}) + 1.7\exp(a\sqrt{s}) + 1)}{(\exp(2a\sqrt{s}) + \exp(a\sqrt{s}) + 1)(\exp(2a\sqrt{s}) - \exp(a\sqrt{s}) + 1)} \tag{9}$$

In fig. 6-7 are shown the magnitude and phase characteristics for eq. (9). Applying the "$s \rightarrow W$" transformation to (9)

$$A_v(W) = \frac{(1-W^2)(W^2+12.33)}{(W^2+3)(W^2+0.333)} = \frac{-y_{21}}{Y_{22} + kW} \tag{10}$$

$y_{21}(W)$ and $y_{22}(W)$ are chosen so that the realizability conditions of Corollary II and the pole-zero alternation for $y_{22}(W)$ are met.

$$-y_{21}(W) = \frac{(1 - W^2)(W^2 + 12.33)}{(W^2 + 1)\ W} \tag{11}$$

$$y_{22}(W) + kW = \frac{(W^2 + 3)(W^2 + 0.333)}{W(W^2 + 1)} \tag{12}$$

$$k = 0.1$$

$$y_{22}(W) = \frac{0.9(W^2 + 3.25)(W^2 + 0.34)}{W(W^2 + 1)} \tag{13}$$

The conditions of Corollary II are met. The zeros of $y_{21}(W)$ contributed by $(W^2 + 12.33)$ and the $(1 - W^2)$ term may be achieved in one extraction cycle. The minimum resistance of a single shunt segment which may be extracted at first is given by (2a) to be

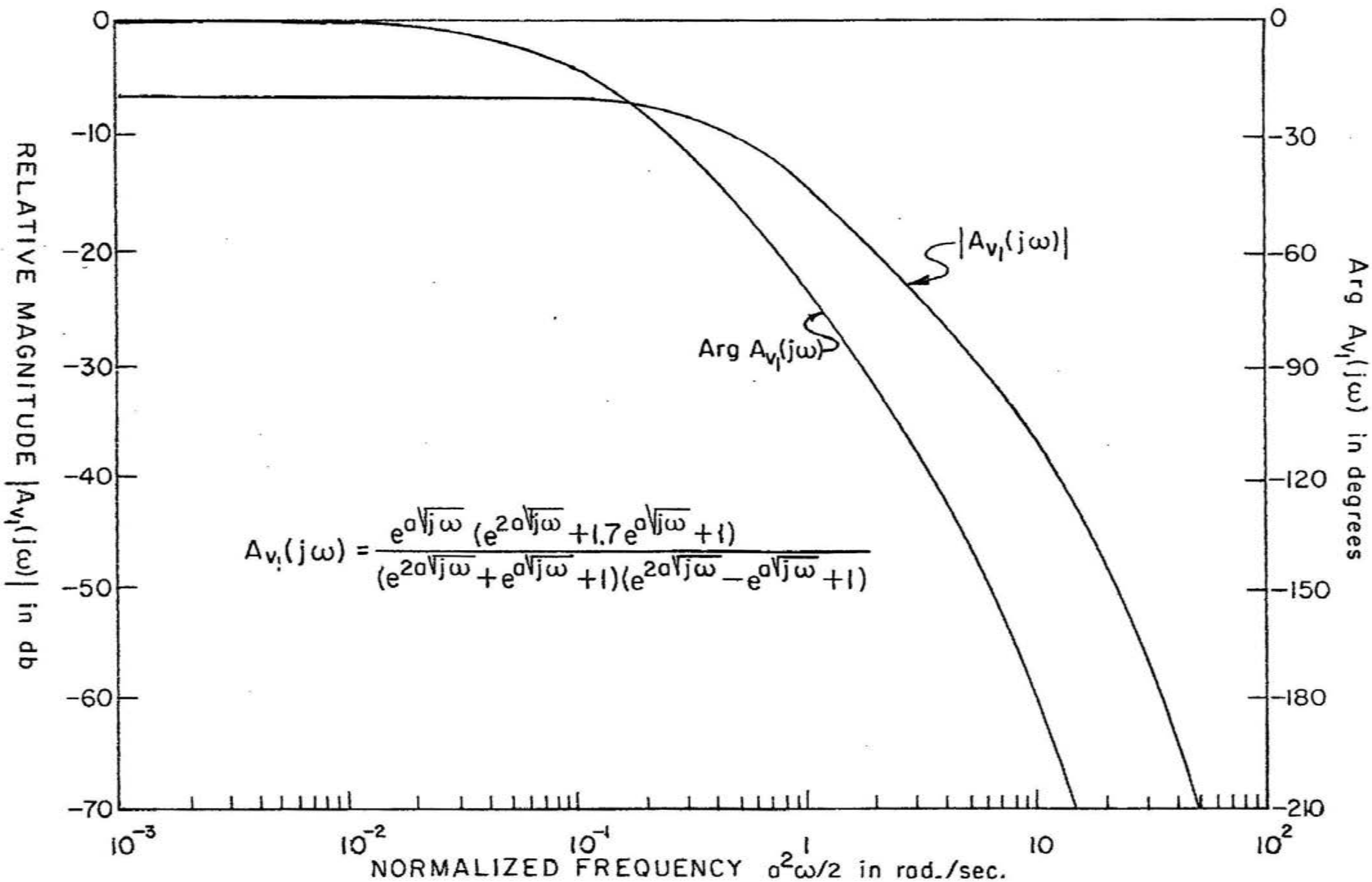

Figure No. 6-7 NETWORK "B" MAGNITUDE AND PHASE CHARACTERISTICS

$$\frac{\sqrt{\tau}}{R_o} = \frac{Y(W)}{W}\bigg|_{W \to \infty} = 1 \tag{14}$$

Applying (3) at the transmission zero, $W = j\sqrt{12.33}$,

$$\frac{\sqrt{\tau}}{R_{o1}} = \frac{y(1) - jQY(jQ)}{1 + Q^2} = \frac{2.85 + 961}{13.33} = 0.935 \tag{15}$$

may be extracted. Doing so

$$Y_1(W) = Y(W) - 0.935\,W$$

$$\frac{0.065W^4 + 2.657W^2 + 1.11}{W(W^2 + 1)} \tag{16}$$

$$Y_1(1) = 1.91 = \frac{\sqrt{\tau}}{R_1} \tag{17}$$

Then applying Richard' s theorem, a cascade section is extracted.

$$Y_2(W) = Y_1(1)\,\frac{Y_1(W) - W\,Y_1(1)}{Y_1(1) - WY_1(W)}$$

$$= \frac{38.83\,(W^2 + 0.602)}{W(W^2 + 12.33)} \tag{18}$$

In accordance with (7), a two-segment stub is used to remove the transmission zeros due to the $(W^2 + 12.33)$ factor:

$$Y_5(W) = \frac{36.93\,W}{W^2 + 12.33} \tag{19}$$

This admittance can be realized as a series connection of one URCO ($R_{03} = 0.34\sqrt{\tau}$) and one URCS ($R_{04} = 0.27\sqrt{\tau}$)

The remaining admittance is

$$Y_3(W) = Y_2(W) - Y_5(W)$$

$$= \frac{38.83(W^2 + 0.602)}{W\,(W^2 + 12.33)} - \frac{36.93W}{W^2 + 12.33}$$

$$= \frac{1.90\,(W^2 + 12.33)}{W(W^2 + 12.33)} = \frac{1.90}{W} \tag{20}$$

$$Y_3(1) = 1.90 = \frac{\sqrt{\tau}}{R_3}$$

$$R_3 = 0.525\sqrt{\tau}$$

$\sqrt{\tau}$ is again chosen for the proper frequency scaling. The final network is shown in fig. 6-8. The physical embodiment is shown in fig. 6-9.

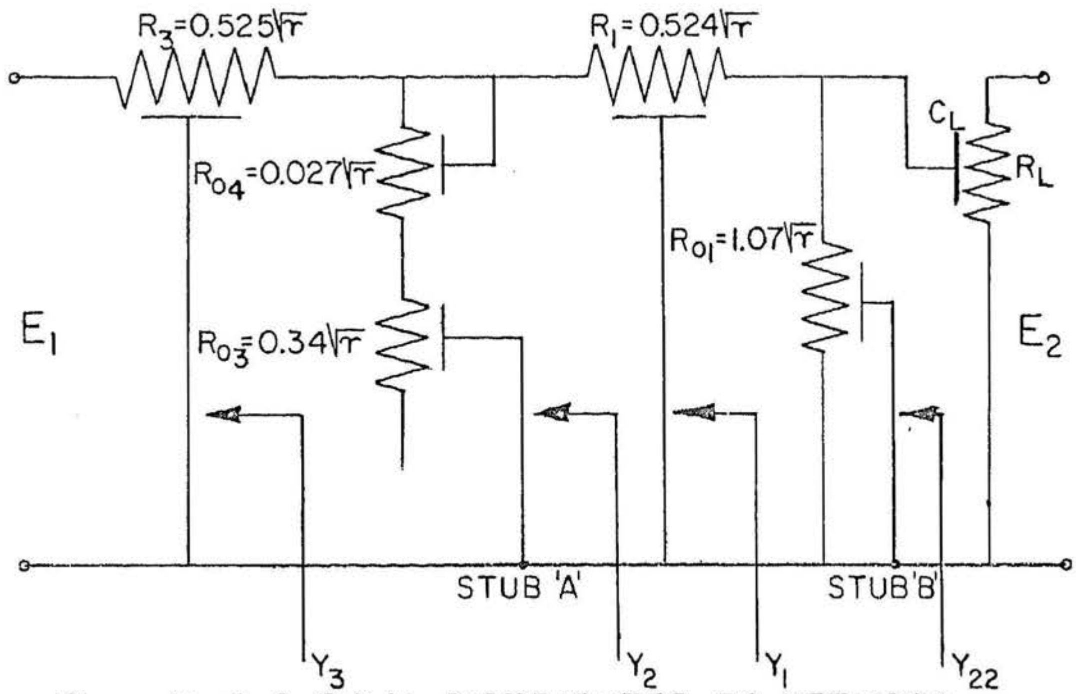

Figure No. 6-8 FINAL DISTRIBUTED RC NETWORK

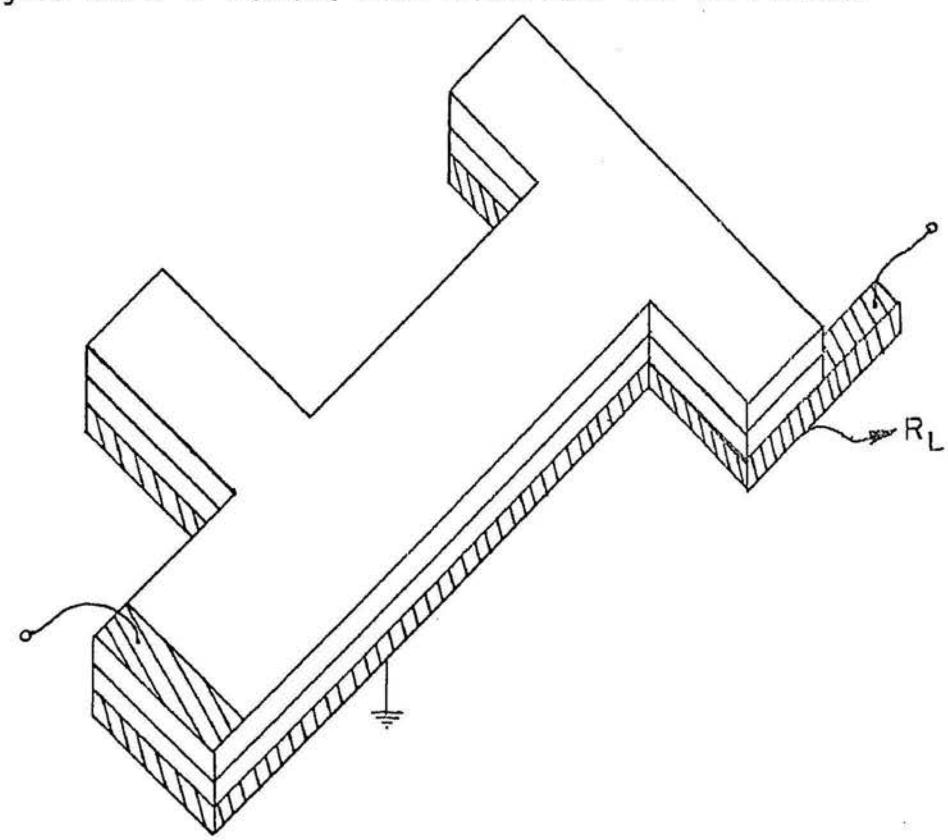

Figure No. 6-9 CASCADE-STUB PHYSICAL EMBODIMENT.

CHAPTER VII

DISTRIBUTED RCR NETWORKS

Because of the distributed RC lines characteristics it is not possible to achieve finite zeros of transmission using those devices. Consequently, another structure to assure negative real transmission zeros is needed. This structure is the distributed RCR network, illustrated in fig. 7-1. The impedance matrix for the network is given by[4]

$$Z(p) = \begin{bmatrix} R_A \dfrac{\text{Coth } nL}{n} & R_A \dfrac{1}{n \text{ Sinh } nL} \\[2ex] R_A \dfrac{1}{\text{Sinh } nL} & R_A \dfrac{\text{Coth } nL}{n} \end{bmatrix} \qquad (1)$$

Where

$$n = \sqrt{\frac{pR_A C}{1 + pC/G_B}} = \sqrt{R_A G_B}\sqrt{\frac{pC/G_B}{1 + pC/G_B}} = k\sqrt{\frac{p'}{1 + p'}}$$

$$k = \sqrt{R_A G_B} \;\; ; \; p' = pC/G_B$$

If a new variable, s, is defined by $s = \dfrac{p'}{1 + p'}$, s is clearly a positive real function of p'.

The realizability conditions for distributed RC networks can be applied to the distributed RCR networks after replacing s by $p'/(1 + p')$. These functions can also be transformed to the W-plane by eqn. 5 of Chapter IV.

The magnitude and phase characteristics for exponential polynomial factors of the complex variable $p'/(1+p')$, for $p' = j\omega$, which are needed for the approximation of these functions, are shown in figs. 7-2 to 7-8.

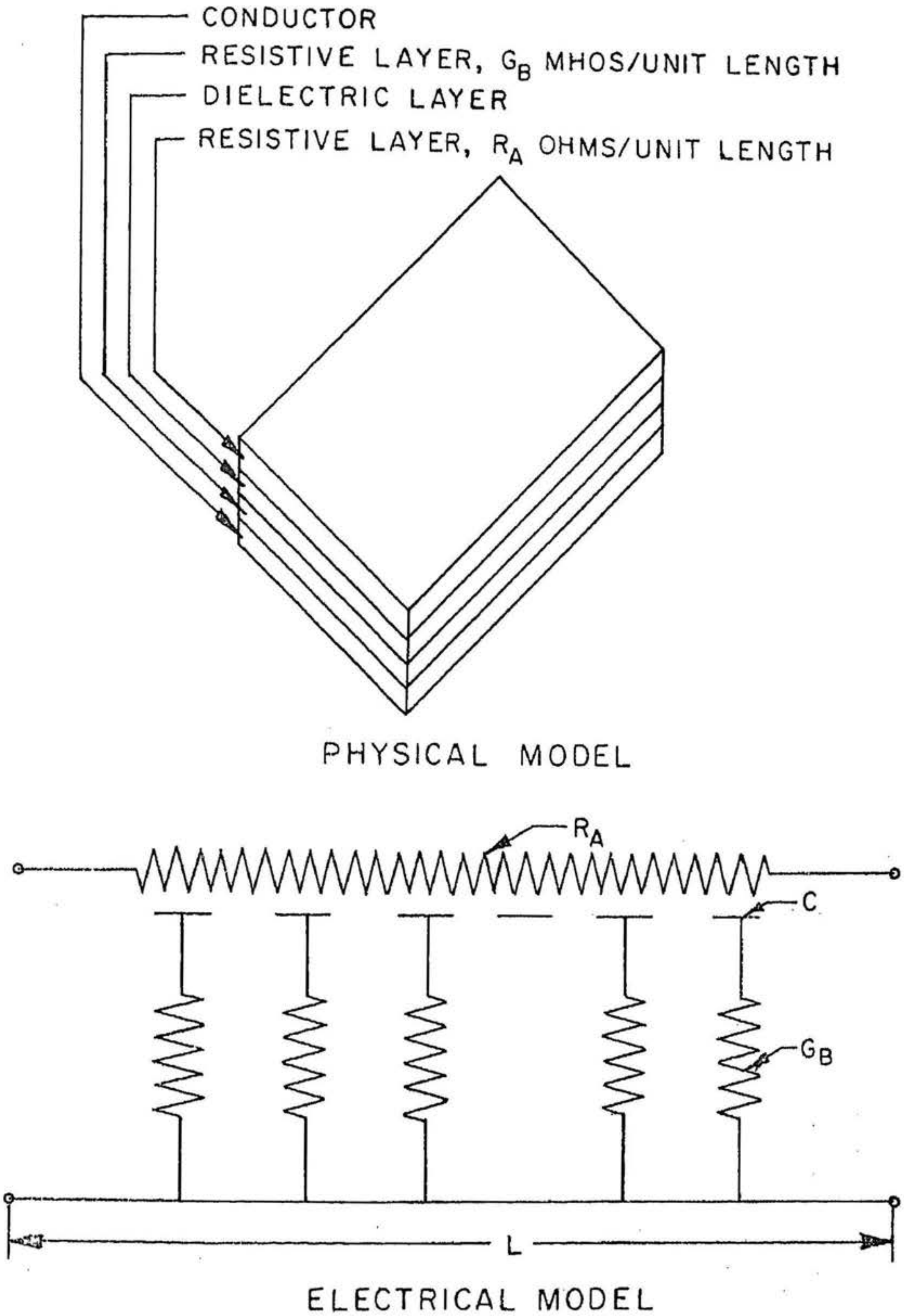

Figure No. 7-1 DISTRIBUTED RCR LINE SEGMENT

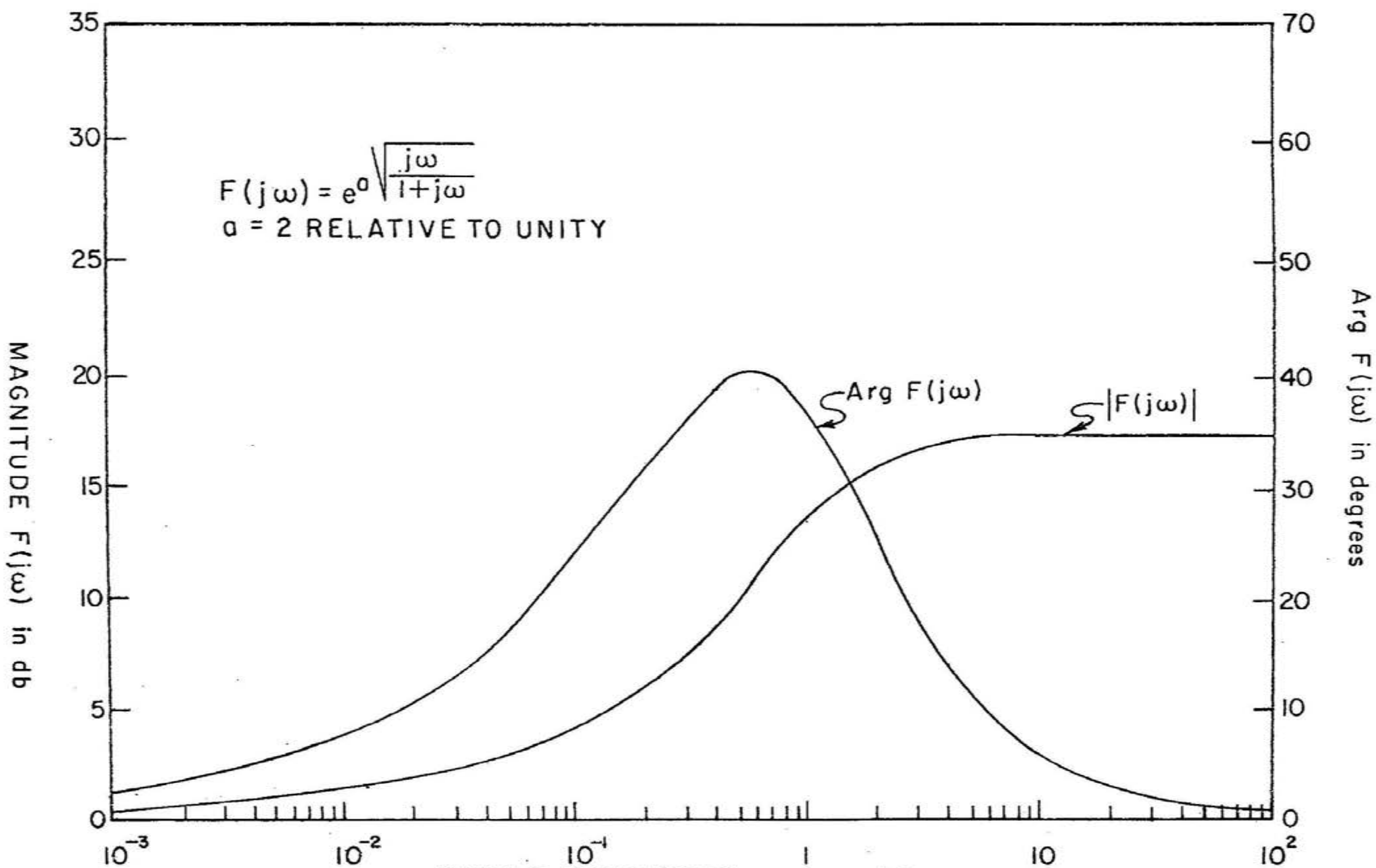

Figure No. 7-2 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RCR SYNTHESIS

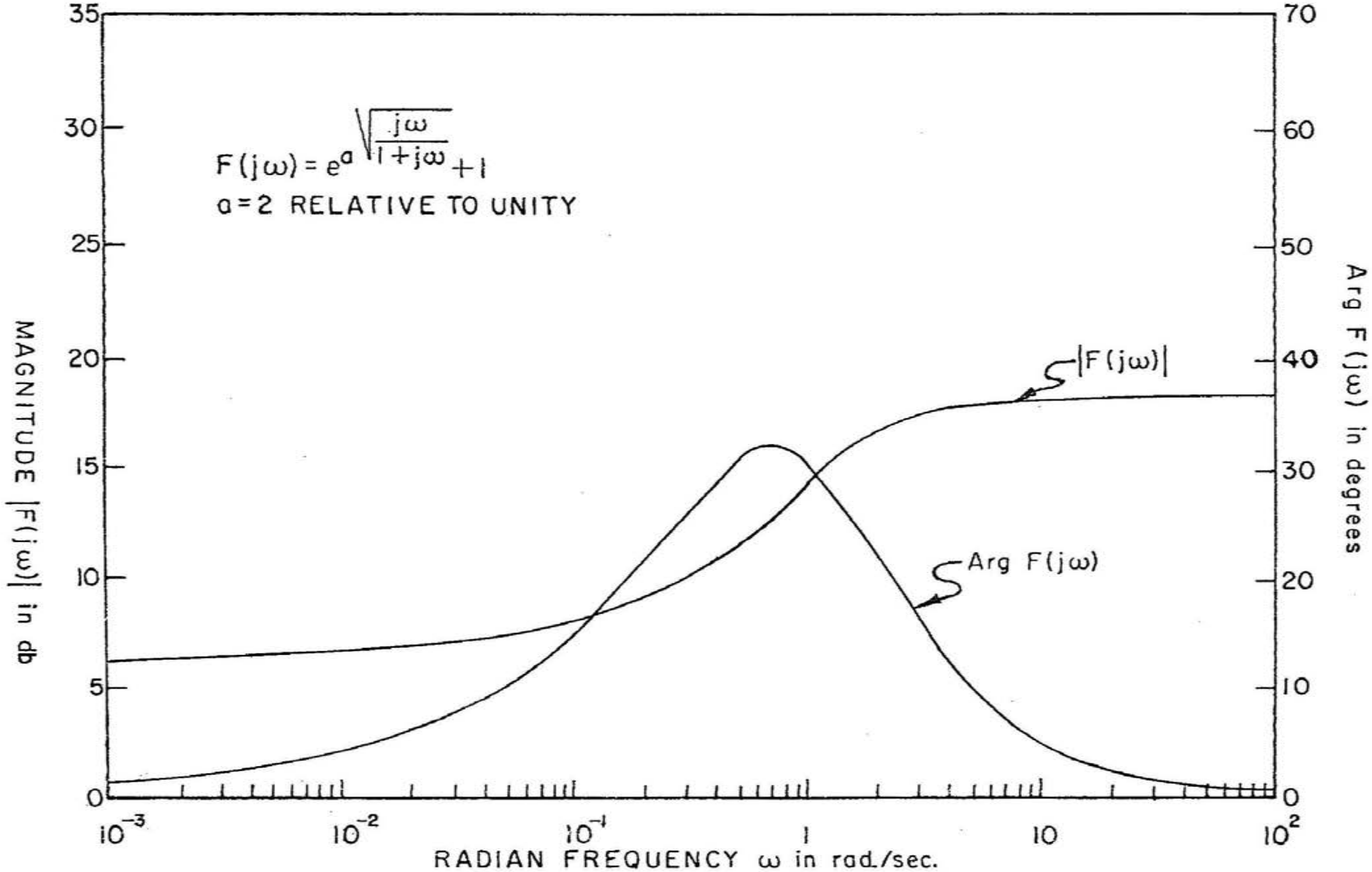

Figure No. 7-3 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RCR SYNTHESIS

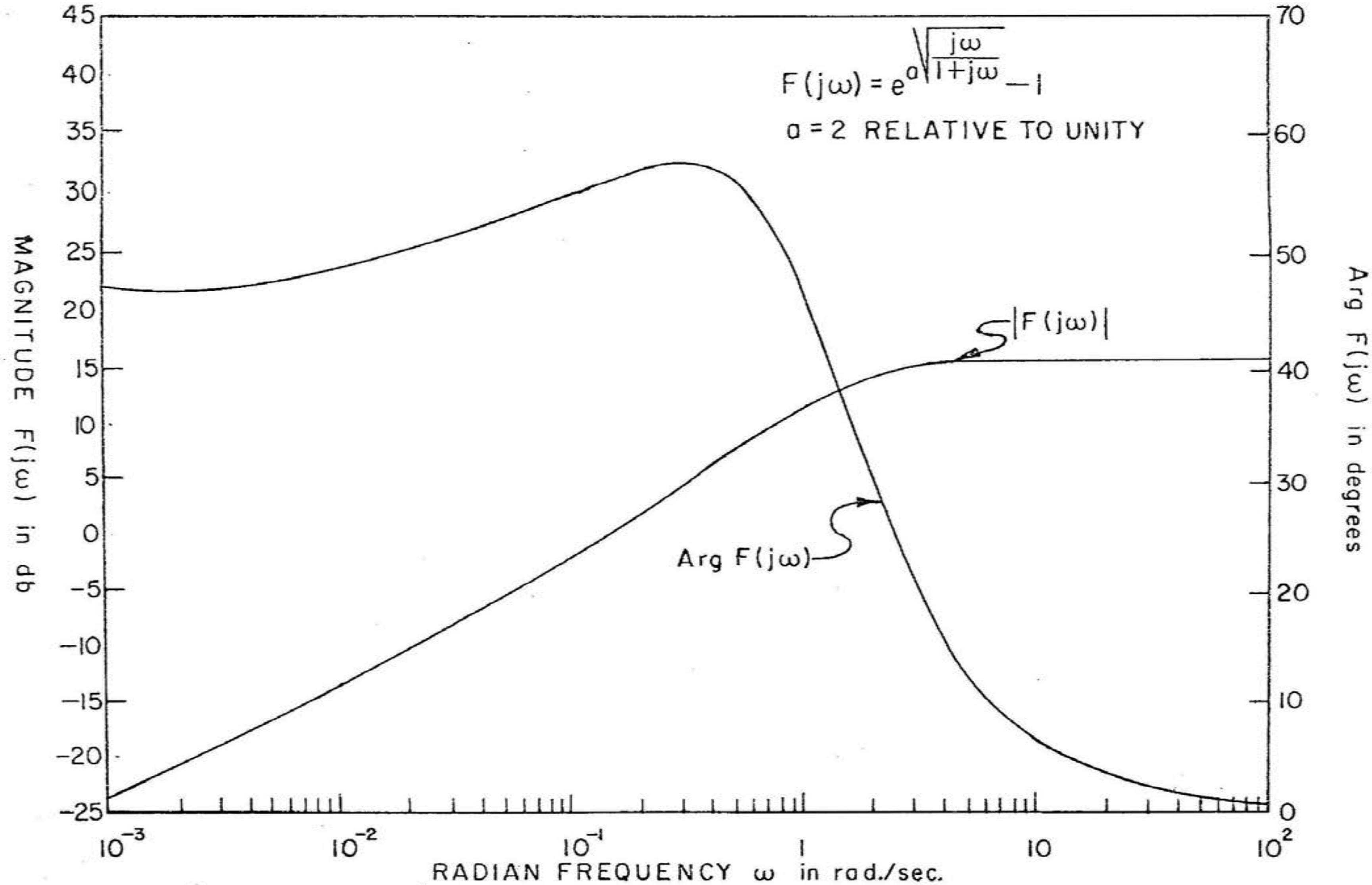

Figure No. 7-4 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RCR SYNTHESIS

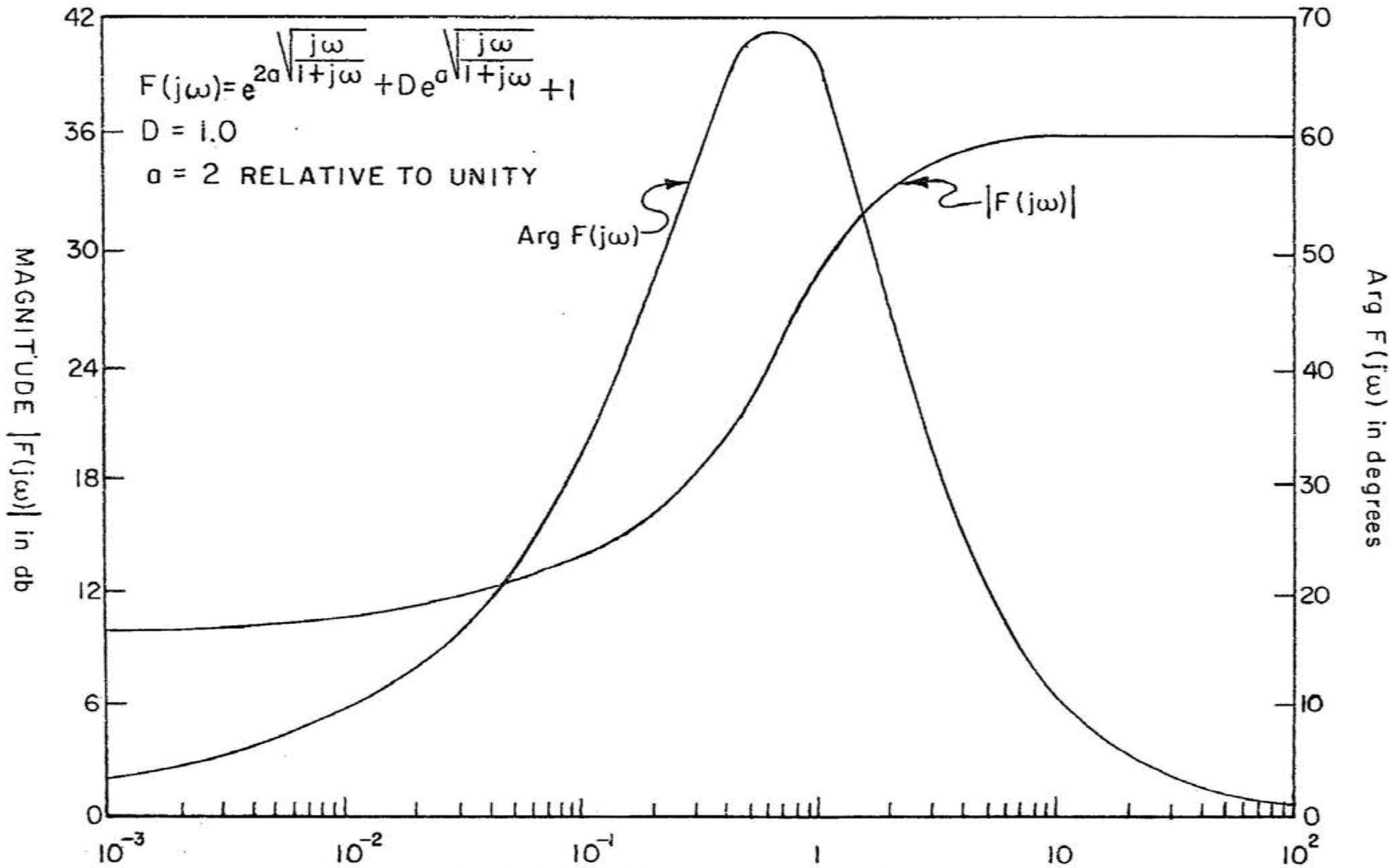

Figure No. 7-5 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RCR SYNTHESIS

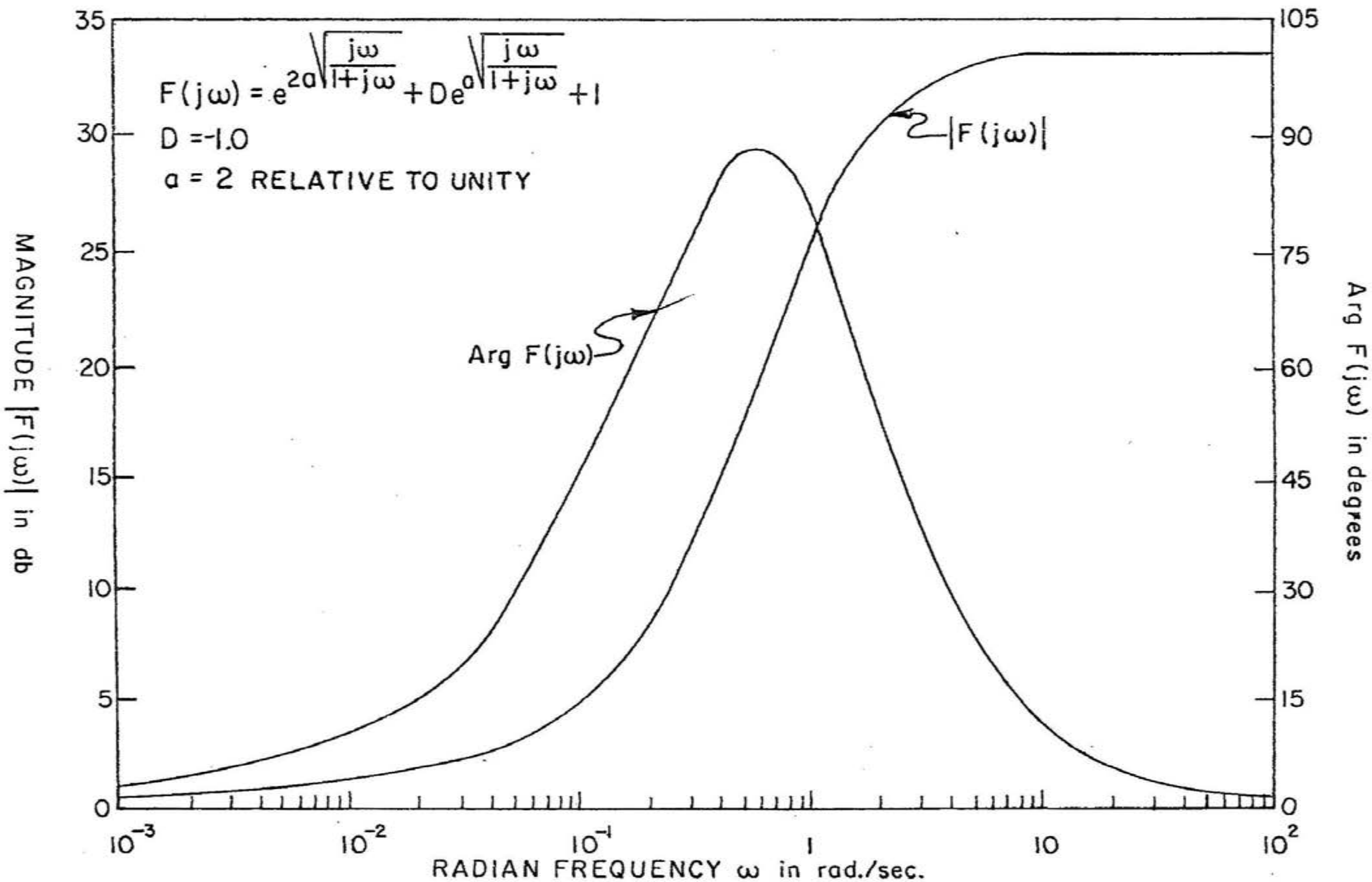

Figure No. 7-6 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RCR SYNTHESIS

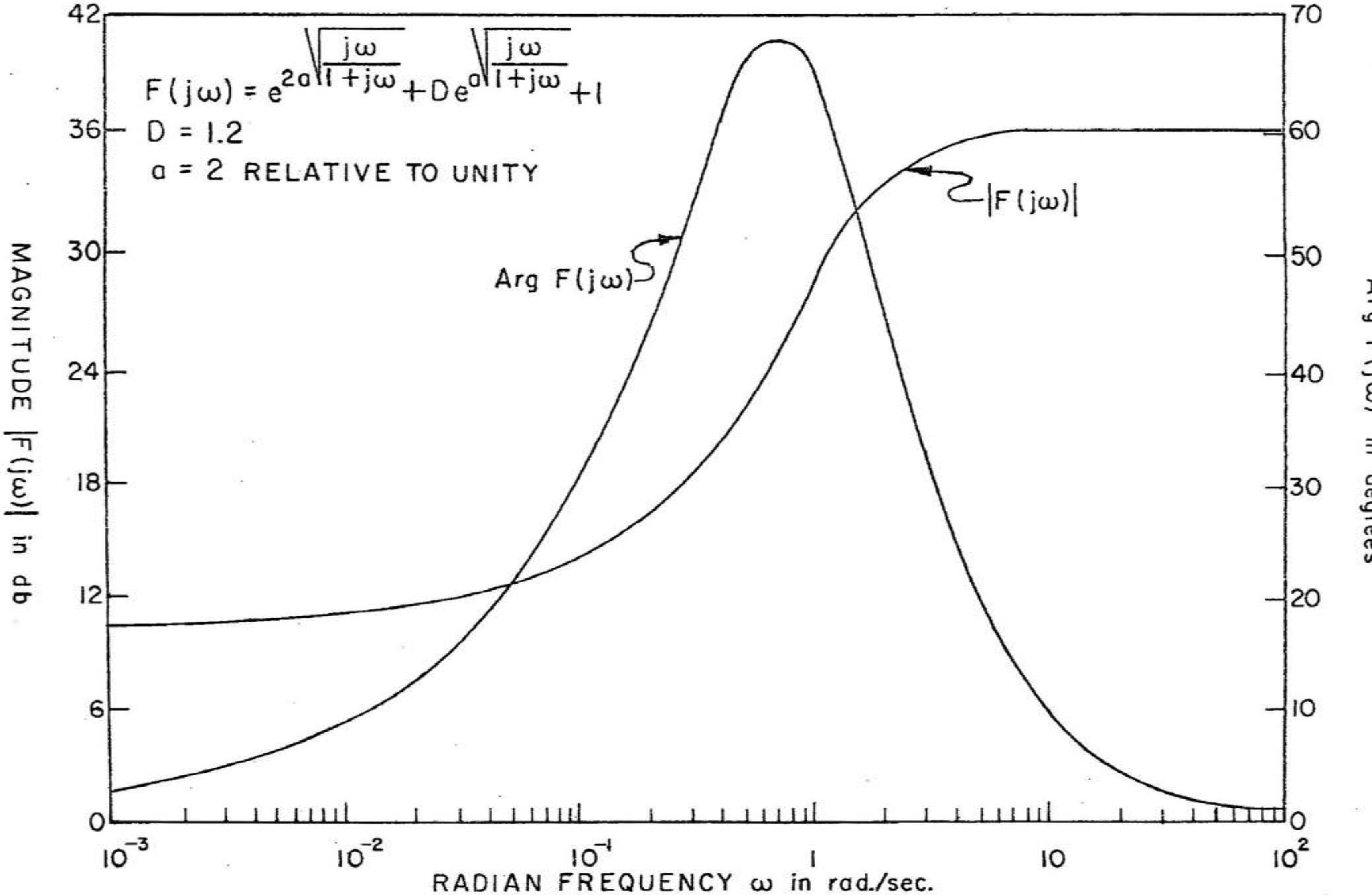

Figure No. 7-7 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RCR SYNTHESIS

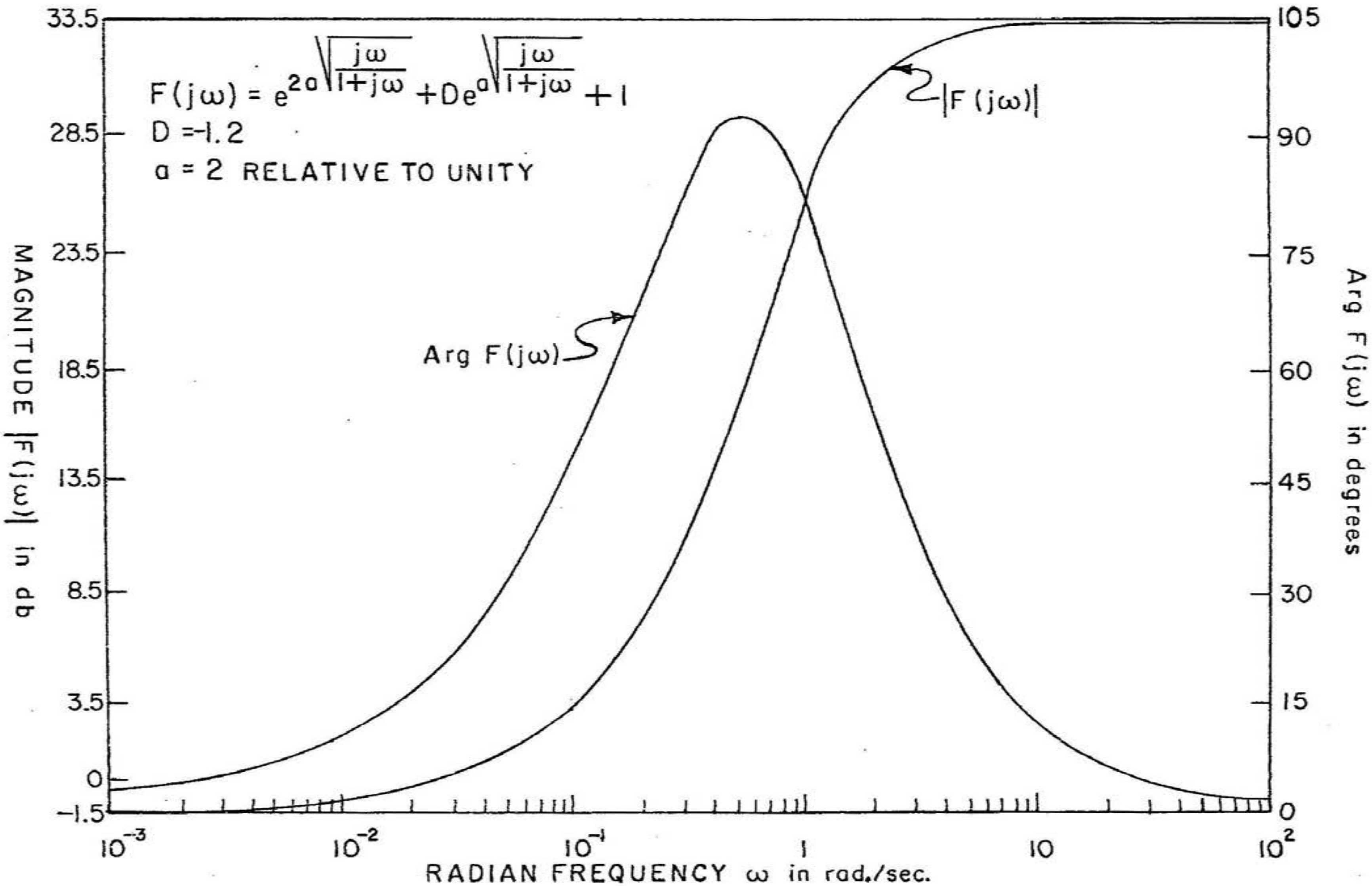

Figure No. 7-8 MAGNITUDE AND PHASE FUNCTIONS FOR DISTRIBUTED RCR SYNTHESIS

Transfer functions that can be used as lag and lead compensators can be obtained by applying the approximation techniques to the exponential polynomial factors.

Eqs. (2) and (3) describe the transfer function for a lag and a lead compensator respectively,

$$Av(p') = \frac{e^{3a\sqrt{p'/(1+p')}}}{(e^{2a\sqrt{p'/(1+p')}}+1.2e^{a\sqrt{p'/(1+p')}}+1)(e^{2a\sqrt{p'/(1+p')}}-1.2e^{a\sqrt{p'/(1+p')}}+1)} \quad (2)$$

$$Av(p') = \frac{e^{4a\sqrt{p'/(1+p')}}}{(e^{2a\sqrt{p'/(1+p')}}+1.2e^{a\sqrt{p'/(1+p')}}+1)(e^{2a\sqrt{p'/(1+p')}}-1.2e^{a\sqrt{p'/(1+p')}}+1)} \quad (3)$$

The magnitude and phase characteristics for these two functions are shown in figs. 7-9, and 7-10 respectively.

By use of the "s $\to$ W" transformation and Richard' s theorem these functions can be synthesized as a cascade of distributed RCR segments as shown in fig. 7-11.

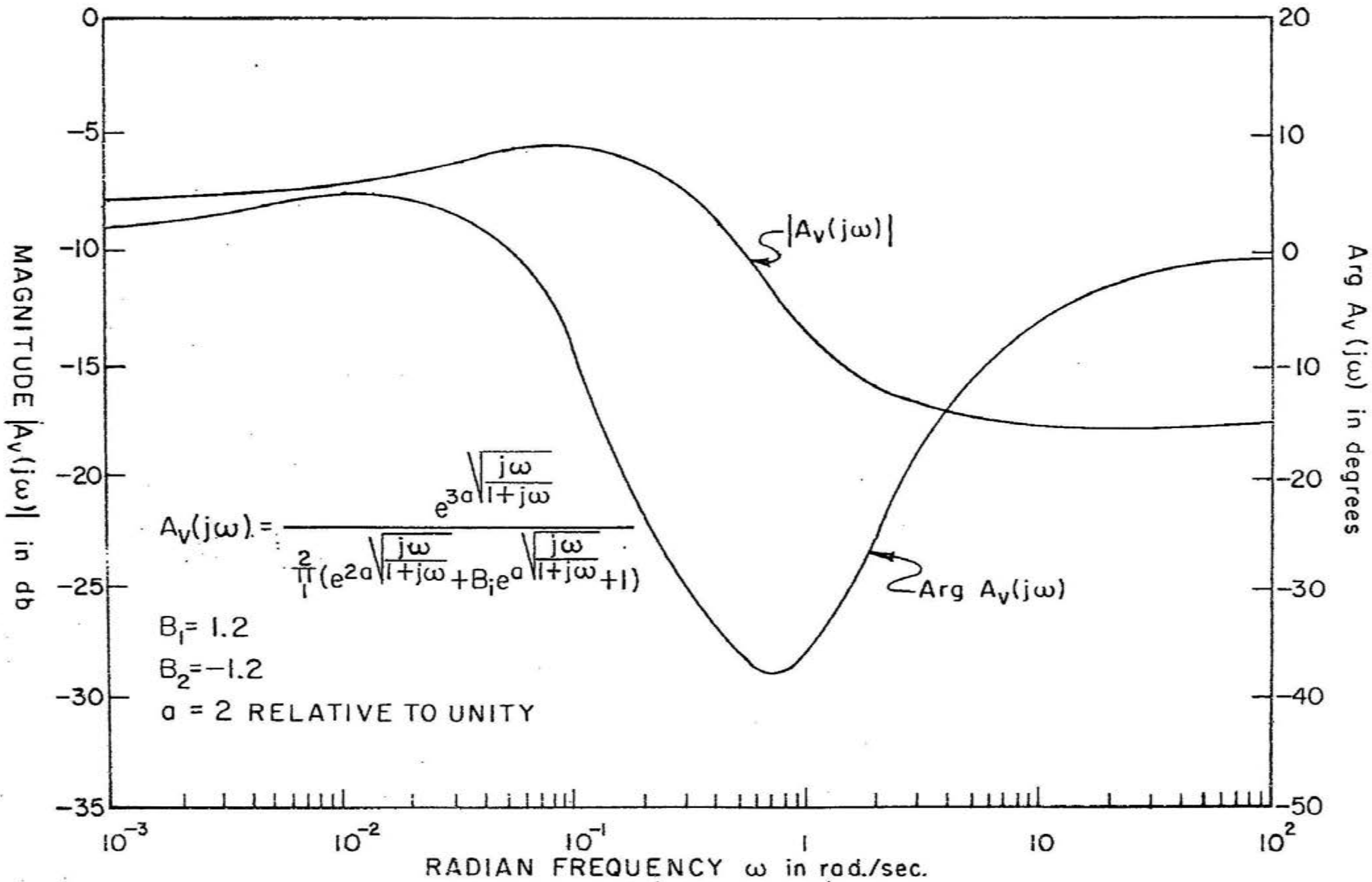

Figure No. 7-9 APPROXIMATION OF $A_v(p^l)$ BY EXPONENTIAL POLYNOMIALS.

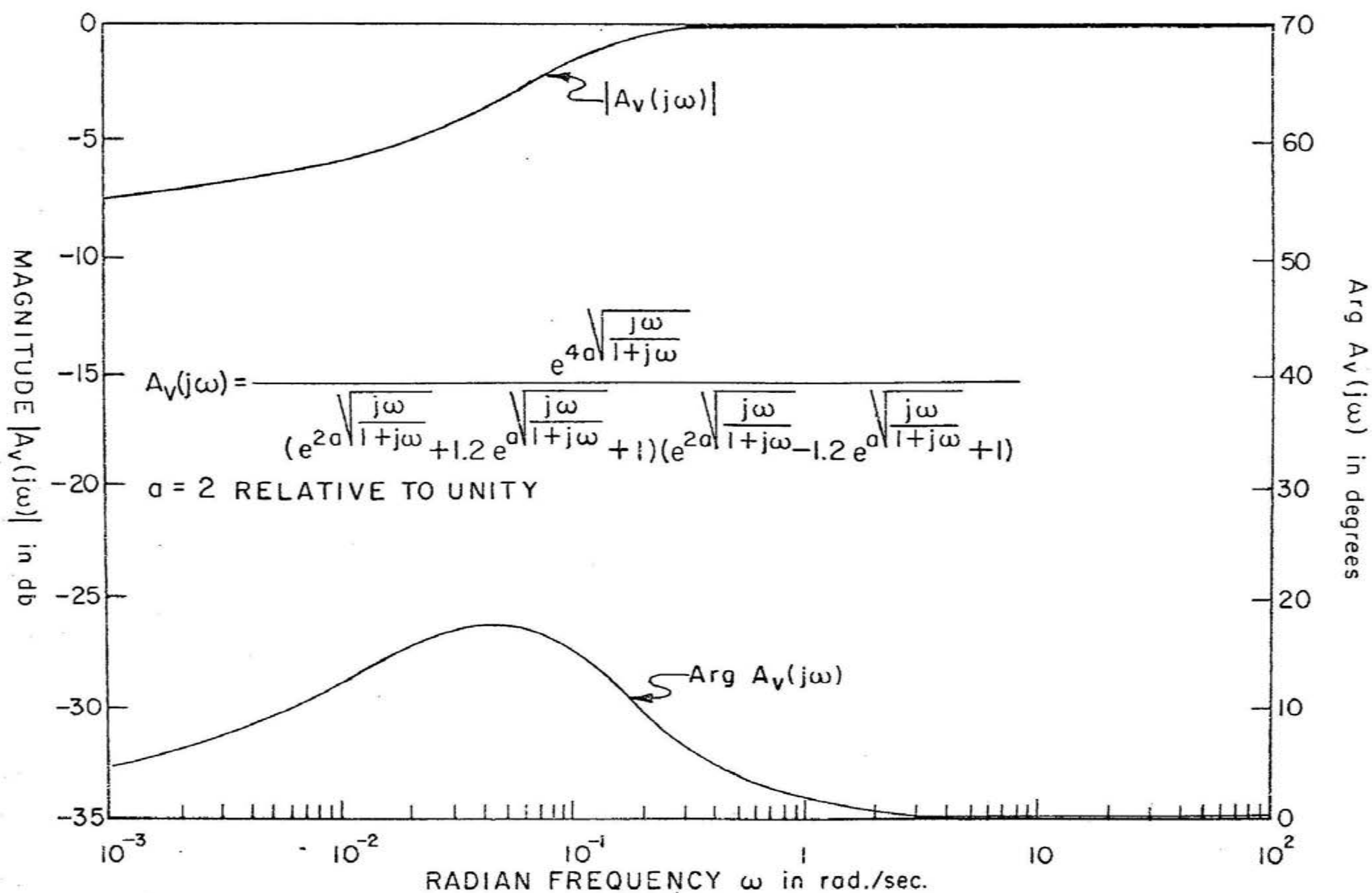

Figure No. 7-10 APPROXIMATION OF $A_V(p^1)$ BY EXPONENTIAL POLYNOMIALS.

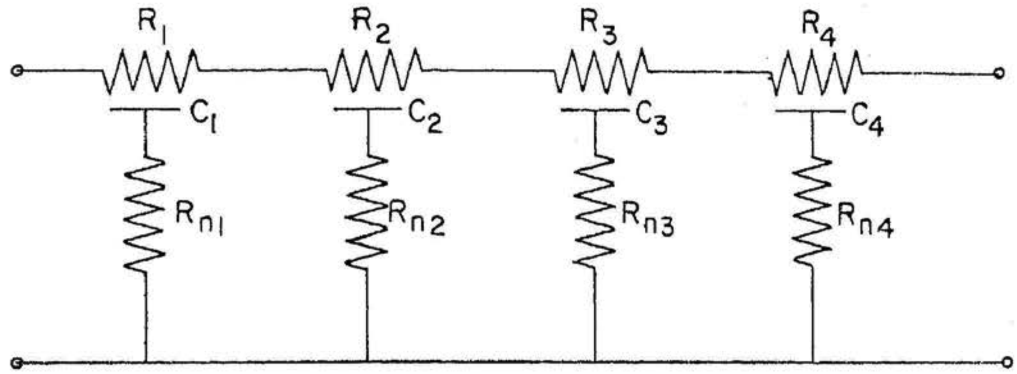

Figure No. 7-11 CASCADE OF DISTRIBUTED RCR SEGMENTS

CHAPTER VIII

CONCLUSIONS

The filter synthesis procedures presented here find immediate application in the field of microelectronics.

A contribution to the distributed RC characterization problem has been developed with the introduction of the "Exponential polynomial factors" approximate frequency magnitude and phase characteristics.

It was shown that the "s $\rightarrow$ W" transformation reduces the synthesis problem associated with distributed RC or RCR networks to that of lumped LC networks.

The cascade configuration used in Chapter IV is practical and attractive from the fabrication viewpoint because it minimizes interconnections.

The ladder configuration of Chapter V has the disadvantage of having too many interconnections which is impractical from the fabrication viewpoint.

The "cascade and stub" method illustrated in Chapter VI was shown to be useful in the synthesis of band-pass filters. It was shown that distributed RCR networks can be used as lag and lead compensators, and are useful when finite zeros of transmission are required.

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VITA

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